

Teacher and student actions

Mathematical discourse among students is central to meaningful learning of mathematics. Teachers carefully prepare and purposefully facilitate discourse, such as whole-class discussions that build on student thinking and guide the learning of the class in a productive disciplinary direction. Students are active members of the discourse community as they explain their reasoning and consider the mathematical explanations and strategies of their classmates. The actions listed in the table below provide some guidance on what teachers and students do as they engage in meaningful discourse in the mathematics classroom.

Facilitate meaningful mathematical discourse Teacher and student actions	
What are <i>teachers</i> doing?	What are <i>students</i> doing?
<p>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</p> <p>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</p> <p>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</p> <p>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</p>	<p>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</p> <p>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</p> <p>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.</p> <p>Identifying how different approaches to solving a task are the same and how they are different.</p>

Pose Purposeful Questions

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematical discourse. Purposeful questions allow teachers to discern what students know and adapt lessons to meet

varied levels of understanding, help students make important mathematical connections, and support students in posing their own questions. However, merely asking questions is not enough to ensure that students make sense of mathematics and advance their reasoning. Two critical issues must be considered—the types of questions that teachers ask and the pattern of questioning that they use.

Discussion

Researchers have created a variety of frameworks to categorize the types of questions that teachers ask (e.g., Boaler and Brodie 2004; Chapin and O'Connor 2007). Though the categories differ across frameworks, commonalities exist among the types of questions. For example, the frameworks generally include questions that ask students to recall information, as well as questions that ask students to explain their reasoning. Figure 14 displays a set of question types that synthesizes key aspects of these frameworks that are particularly important for mathematics teaching. Although the question types differ with respect to the level of thinking required in a response, all of the question types are necessary in the interactions among teachers and students. For example, questions that gather information are needed to establish what students know, while questions that encourage reflection and justification are essential to reveal student reasoning.

Question type		Description	Examples
1	Gathering information	Students recall facts, definitions, or procedures.	When you write an equation, what does the equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?
2	Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	As you drew that number line, what decisions did you make so that you could represent $\frac{7}{4}$ on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?

Fig. 14. A framework for types of questions used in mathematics teaching

Question type		Description	Examples
3	Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	<p>What does your equation have to do with the band concert situation?</p> <p>How does that array relate to multiplication and division?</p> <p>In what ways might the normal distribution apply to this situation?</p>
4	Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	<p>How might you prove that 51 is the solution?</p> <p>How do you know that the sum of two odd numbers will always be even?</p> <p>Why does plan A in the Smartphone Plans task start out cheaper but become more expensive in the long run?</p>

Fig. 14. *Continued*

While the *types* of questions that teachers ask are important, so are the *patterns* of questions that they use during teacher-student interactions (Walsh and Sattes 2005). In the Initiate-Response-Evaluate (I-R-E) pattern, the teacher starts by asking a question to gather information, generally with a specific response in mind; a student responds; and then the teacher evaluates the response (Mehan 1979). It is not uncommon for teachers to allocate less than five seconds for a student to respond, and to take even less time to consider the answer themselves. This pattern of questioning generally affords very limited opportunities for students to think and provides teachers with no access to whether or how students are making sense of mathematics. Other questioning patterns involve more than asking recall questions. Two of these patterns of questioning are *funneling* and *focusing* (Herbel-Eisenmann and Breyfogle 2005; Wood 1998).

The funneling pattern of questioning involves using a set of questions to lead students to a desired procedure or conclusion, while giving limited attention to student responses that veer from the desired path. The teacher has decided on a particular path for the discussion to follow and leads the students along that path, not allowing students to make their own connections or build their own understanding of the targeted mathematical concepts. The I-R-E pattern is closely akin to funneling, though higher-level questions may be part of the funneling pattern.

In contrast, a focusing pattern of questioning involves the teacher in attending to what the students are thinking, pressing them to communicate their thoughts clearly, and expecting them to reflect on their thoughts and those of their classmates. The teacher who uses this pattern of questioning is open to a task being investigated in multiple ways. On the basis of content knowledge related to the topic and knowledge of student learning, the teacher plans questions and outlines key points that should become salient in the lesson.