CPM EDUCATIONAL PROGRAM

CPM's 2023 Research Base
Executive Summary
Mixed, Spaced Practice

This is a summary of CPM's 2023 research base on Mixed, Spaced Practice. For more information and for references, please see the full report, available from https://cpm.org/research-base/.

What is Mixed, Spaced Practice?

Because research uses the term "interleaving" to address what CPM has historically called Mixed, Spaced Practice, this document uses interleaving when summarizing research on Mixed, Spaced Practice. *Interleaving* is a hybrid of mixed and spaced problems: interleaved mathematics problems involve *mixed* problem sets with similar types of problems *spaced* over multiple weeks or months.

CPM's Pillar of Mixed, Spaced Practice refers to practice in which:

A. the order of problems is randomized so that students do not know which type of problem will come next; and
B. students' opportunities to encounter particular problem types are spaced out over days, weeks, and even years.

Mixed, Spaced Practice is a pillar of CPM that long predates studies on interleaving in mathematics education. Mixed, Spaced Practice is visible in CPM materials in both independent practice (e.g., Review & Preview or Reflection & Practice) and in threaded core content throughout units, courses, and course sequences. As students reencounter problem types over time (within a course and across courses), the problems increase in complexity.

Overview

This document summarizes recent studies in which researchers have found overwhelming evidence for the strong benefits of increased retention (long-term memory) and transfer (recognizing problem types in unfamiliar problems) when students engage in interleaved practice rather than blocked and massed practice.

CPM infers that…

Mixed, Spaced Practice supports students in developing mathematical dispositions in which they strive to make sense of problems rather than seek correct answers by mindlessly following procedures. Students will develop procedural fluency as they become increasingly proficient at identifying problem types and selecting appropriate problem-solving strategies.

Why have I not encountered this before?

In a study of six commonly adopted textbooks — Big Ideas Math, Connected Mathematics, Glencoe Math: Built to the Common Core, Go Math, Holt McDougal Mathematics, and SpringBoard Mathematics — Rohrer et al. (2020a) found that all of these textbooks predominantly contained blocked practice. They concluded their report with a call to action.

> We recommend that mathematics teachers avoid assigning long blocks of practice problems. No direct evidence suggests that students benefit from solving more than a few problems of the same kind in immediate succession, whereas interleaved assignments are supported by numerous classroom-based randomized controlled studies. (Rhohrer et al., 2020a, p. 881–882)

CPM infers from this research that…

Mixed, Spaced Practice is a proven approach to developing procedural fluency. Unlike the proven positive effects of Mixed, Spaced Practice, there is no direct empirical evidence that worksheets or technology requiring students to solve many problems of the same type in close succession benefit students' learning in the long-term. In fact, researchers actively recommend that teachers do not give blocked or massed practice problems and instead give Mixed, Spaced Practice.

CPM is unique[1] in its built-in design for Mixed, Spaced Practice, lessening the load on teachers to resequence problems as they would need to do with many other textbooks. We encourage CPM teachers not to rearrange content but to rely on the evidence-based design of CPM's threaded content in core problems and Mixed, Spaced Practice in Review & Preview / Reflection & Practice. Mixed, Spaced Practice works for supporting students' long-term retention and transfer of mathematics concepts and skills. In the remainder of this document, we summarize research that justifies this claim.

Who is Mixed, Spaced Practice good for?

Yan and Sana (2021) conducted a study that examined the effects of interleaving on (a) learning in both low- and high-complexity tasks and (b) how that learning varied across learners with low and high working memory capacity. These researchers were unable to identify a condition in which blocking was more effective for learners with either low or high working memory capacity.

CPM infers from this research that…

Mixed, Spaced Practice is beneficial for most students when solving problems with or without rich contexts. Thus far, research indicates that randomizing problem order is always more effective for learning mathematics than grouping together and sequencing problem types.

[1] Saxon is well known for its interleaving. However, Saxon was not on adoption lists at the time of the study cited, perhaps because it had not been updated to fit current frameworks.

What are some benefits of Mixed, Spaced Practice?

Increases achievement

In a randomized control trial of 54 seventh-grade mathematics classes (15 teachers total), Rohrer et al. (2020b) found that students who completed interleaved practice assignments outscored students who completed blocked practice assignments. Even when the conditions of the test better matched students in the blocked practice condition than the interleaved practice condition, students who completed interleaved practice sets scored much better on the posttest than did students who completed blocked practice assignments.

CPM infers from this research that…

Mixed, Spaced Practice supports markedly higher achievement on assessments than blocked practice. This is likely true for standardized assessments as well, since Mixed, Spaced Practice allows students to repeatedly encounter problem types throughout an academic year.

Supports long-term retention

The benefits of interleaving become clear when students are assessed at time intervals that indicate their long-term retention rather than when students are assessed shortly after they have practiced particular problem types. For example, Rohrer et al. (2020b) found that the retention benefits of interleaving over blocking practice (i.e., effect sizes) became less pronounced on problem types that the blocked practice group experienced closer to the test. Even more, only one study (Ostrow et al., 2015) has ever found that interleaving did not produce stronger learning outcomes than blocked practice, and this study used a very brief test delay of 2-5 days. Taken together, these findings indicate that interleaved practice supports long-term retention at least as well as — but likely much better than — blocked practice. Very importantly, Rohrer et al. (2020b) note that interleaved practice is likely only effective if students receive meaningful feedback.

CPM infers from this research that…

Mixed, Spaced Practice is an effective way to support long-term retention and build enduring procedural fluency. The fact that Mixed, Spaced Practice is likely only effective if students receive meaningful feedback is not an indication that homework or other independent practice should be graded. In fact, some research suggests that grading homework perpetuates inequities (Feldman, 2018). Instead, CPM interprets this research on Mixed, Spaced Practice to indicate that students learn best when they have opportunities to learn from their mistakes, which can be accomplished through self-assessment and peer feedback, among other strategies.

Supports transfer on procedural problems and word problems

Interleaving is useful not only for procedural problems but is also beneficial for more complex mathematical tasks. For example, Sana, Yan, & Kim (2017) found that students were better able to correctly categorize different types of statistics word problems when their tasks leading up to the experiment were interleaved.

CPM infers from this research that…

Mixed, Spaced Practice is beneficial throughout mathematics textbooks rather than only in procedural and independent practice. In CPM materials, Mixed, Spaced Practice at the course level is described in terms of *content threads* that weave throughout each course and across courses to support connections and retention.

Provides robust support for learning

In a 2022 study of the conditions under which interleaving was effective, Mielicki and Wiley found that there was a marginal positive difference in delayed tests scores for groups that engaged in hybrid practice (e.g., blocked practice before interleaved practice or interleaved practice before blocked practice) over groups that engaged in all interleaved practice. The authors concluded that "hybrid schedules are beneficial relative to entirely blocked practice but may not necessarily confer additional benefits over entirely interleaved practice" (p. 6).

CPM infers from this research that…

Mixed, Spaced Practice has the potential to significantly increase students' retention and transfer and is unlikely to harm students' learning. This research found that students who had low accuracy on Mixed, Spaced Practice assignments did about the same on the assessment as students who scored well on their blocked practice assignments. Students who did well on their Mixed, Spaced Practice assignments scored the best on the assessment of any group. This means that learning through Mixed, Spaced Practice is robust, and ensuring that students get meaningful feedback is important for maximizing its benefits.

What are some trade-offs of implementing Mixed, Spaced Practice?

Learning takes effort: Trade-offs between efficiency and effectiveness

Students in Rohrer et al.'s (2020b) study inaccurately thought they were learning more when problems were easier for them to solve (i.e., lower perceived effort). In fact, even though students in the interleaved condition learned more, they perceived themselves to be learning less and exerting more effort than those in the blocked condition. This indicates that students inaccurately assessed their learning and had misleading views about the relationship between effort and learning. However, according to the researchers, "students were willing to embrace the effort induced by interleaved practice once they recognized that their effort paid off in terms of learning gains" (p. 23). Scholars have also found that massed practices creates a false illusion for both students and teachers that it increases the efficacy of practice, thus leading students to be overconfident (Bjork et al., 2013; Emeny et al., 2021).

CPM infers from this research that…

Though Mixed, Spaced Practice may require more from both students (time, effort) and teachers (feedback), it supports students in developing a learning orientation (i.e., a focus on understanding) rather than a performance orientation (i.e., a focus on correct answers and grades) when it is accompanied by explicit conversations about its benefits, including long-term learning and improved problem-solving skills. This combination of Mixed, Spaced Practice and explicit conversations about

how it pays off over time can help combat the overconfidence that research shows is induced by blocked and massed practice. Research shows that students are much more willing to embrace the increased effort that Mixed, Spaced Practice requires when they understand the significant payoffs for their learning. Students are unlikely to notice these payoffs without support from their teacher.

Learning takes time: Trade-offs between short-term and long-term performance

Though it is clear that interleaving has long-term positive effects on retention, research shows that interleaving may temporarily decrease performance during student work on practice assignments such as homework and quizzes. A study of calculus students' learning found that interleaving practice resulted in better scores on an end-of-semester exam but lower scores on the first two out of three quizzes given, with the last quiz given being unaffected in scores (Lyle et al., 2022). Thus, interleaving makes learning more protracted, but also more robust. The authors of the study pointedly described the effect of interleaving as imposing a "desirable difficulty" in mathematics learning.

CPM infers from this research that…

Though Mixed, Spaced Practice may result in lower scores than blocked or massed practice on assessments given shortly after a new concept is introduced, it does support higher performance long-term, such as on end-of-year exams. This research serves as a reminder that *learning takes time* and is always unfinished. The fact that Mixed, Spaced Practice can lead to lower scores on assessments that occur close to a concept's introduction has implications that align with CPM's longstanding position on assessment: that concepts and skills should be summatively assessed for a grade only after students have had the opportunity to encounter them multiple times in meaningful ways, not only in the unit in which they are first introduced. This does not mean that teachers should not give quizzes, but rather, that quizzes should be considered formative assessments and include meaningful feedback.