



— Executive Summary —

## Intervention and inclusion: Understanding “disability” in the mathematics classroom

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This research brief draws primarily from qualitative research on learning disabilities and mathematics. Importantly, qualitative research is not designed to make claims about causality, meaning that this research does not make scientific claims about what works at scale, but rather is meant to inform how and why particular teaching-and-learning interactions support particular students’ engagement and learning. This brief is not an exhaustive review of existing research, but rather draws purposefully to answer the question:

*How can we support students with learning and intellectual disabilities to experience productive struggle during collaborative problem-solving on cognitively-demanding tasks?*

### An ideological question: What is disability?

This paper uses the phrase “students with identified exceptionalities” for its qualities of putting the person rather than the difference first (“students ... with”), for qualifying that identity-labels are socially constructed and not inherently important (“identified ...”), and for pointing to difference rather than deficit (“exceptionalities”). This new label does not minimize the challenges faced by students with identified exceptionalities. It reframes and relocates the source of these challenges.

The source of exceptionality is not located within individuals; it is located within social institutions and processes, including the physical and social environment. Instead of remediating individuals with neurological differences, we must create greater accessibility such that neurological differences cease to be dis-abling. Dis-ability is not inherent nor static; it is socially constructed in particular teaching contexts (Lambert, 2015). This paper provides conceptual and practical resources for reframing “disability.”

## What counts as inclusion?

Here, *inclusion* does not only mean keeping student bodies in the classroom (i.e., inclusion is not only about place), but it also means providing all students with effective instruction. In this way, this paper addresses equitably teaching students with identified exceptionalities in the general education classroom without trivializing students’ experience of dis-ability. This paper does not take a stance on whether *full* inclusion is appropriate in all contexts.

## Students’ experiences of dis-ability

Alongside experiences of being dis-abled, students often experience secrecy around their diagnosis (Lampert et al., 2019; Rexroat-Frazier & Chamberlin, 2019; Vaughn & Klingner, 1998). This secrecy is detrimental because it creates fixed, shame-ridden mindsets and obfuscates students’ ability to advocate for themselves around their specific learning needs and goals.

Successful individuals with identified exceptionalities lament the endemic educational emphasis on their deficits, arguing that attempts to remediate them to the “average” learner happened at the cost of fostering their strengths (e.g., Lewis & Lynn, 2018; Robinson, 2016; Roy, 2015).

Education *can* foster these strengths. Opportunities to engage in conceptual learning can open up possibilities for students and help them see themselves as capable despite the challenges of neurological differences, such as with memorization.

Notably, Klinger & Vaugh (1999) found that students labelled with learning disabilities tend to prefer the same activities, homework, books, grading, and grouping as their peers without similar labels (Rexroat-Frazier & Chamberlin, 2019). They also found that these same students valued clear explanations, experiencing content in multiple ways, and responsive lesson pacing. Arguably, these are features of teaching and learning that all students might value, with or without identified exceptionalities.

## Importance of multiple perspectives

While the quantitative research that is incredibly prominent special education research is helpful for identifying the persistent errors students make, it falls short of explaining *why* students make particular errors and *why errors persist* despite instruction (Lewis, 2016, p. 100). There is a small but growing body of qualitative research in mathematics education that is beginning to fill this gap.

When examining student thinking, qualitative research focuses on the ways that students *do* understand mathematical concepts and representations instead of only the ways that they do not understand them (i.e., their error patterns). For example, Lewis (2016) identified the persistent understandings of two girls with identified exceptionalities during

fraction comparison tasks (e.g., Which is bigger,  $\frac{2}{8}$  or  $\frac{5}{8}$ ?). By analyzing what sense the students were making rather than only what errors they made, Lewis’ analysis not only portrays the students as sensible doers and thinkers of mathematics, but also provides a foundation to build on in order to move students beyond their current understanding of fraction representations.

Qualitative research has shown that standards-based mathematics curricula — such as CPM — can be made accessible to students with identified exceptionalities (Lambert & Sugita, 2016, p. 362). Because these students can participate in standards-based mathematics, the development of research-based assessments for middle and high school mathematics concepts for students with identified exceptionalities is urgently needed to supplement the existing skills-based assessments of special education.

## Appropriateness of standards-based curricula<sup>1</sup>

Because standards-based mathematics curricula *can* be made accessible to students with identified exceptionalities, it also *should* be made accessible to these students. According to the *Individuals With Disabilities Education Improvement Act* (IDEA, 2004), students with identified exceptionalities should be “To the maximum extent appropriate ... educated with children who are nondisabled” except if “education in regular classes with the use of supplementary aids and services cannot be achieved satisfactorily” (IDEA Sec. 300.114). In other words, all students have the right to inclusive education in the “least restrictive environment” with their peers. More recently, the Supreme Court ruling *Endrew F. v. Douglas County School District* created the precedent that the word *appropriate* in IDEA means *appropriately ambitious*, in that students with identified exceptionalities have the right to meet challenging objectives and fulfil their potential for growth (Wehmeyer, 2019; Wehmeyer, Shogren, & Kurth, 2020).

Unfortunately, as students are given increasingly fewer opportunities for participation in general education classrooms as they progress through the system (Cook & Cook, 2020). This problem is exacerbated for Black and Brown students, who are disproportionately excluded from inclusive settings (Cook & Cook, 2020, p. 137, citing Skiba et al., 2006; Sullivan, 2011).

While Black and Brown students are disproportionately excluded from inclusive settings, educators must (a) take care that white children do not receive better educational “healthcare” due to higher levels of responsiveness to white parents’ concerns and (b) account for the possibility that minoritized students’ parents may be more hesitant to seek out or accept a disability diagnosis due to the historic and experienced marginalization and criminalization of Black and Brown bodies in school buildings (Gregory et al., 2010; Guerrero, Rodriguez, & Flores, 2011; Morgan, Staff, Hillemeier, Farkas, & Maczuga, 2013).

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<sup>1</sup> Standards-based curricula have the following qualities (Jitendra, 2013):

1. Mathematics is encountered through problem solving.
2. Mathematics is embedded in contexts such that mathematical strategies and topics are connected to real-world applications.
3. Mathematics emerges through collaborative teamwork and with mathematical tools (e.g., algebra tiles, calculators).
4. Mathematics begins with student-invented strategies rather than standard algorithms.

## Inclusion does no harm

Importantly, inclusive education does not harm the achievement of students without special educational needs. In a meta-synthesis of 47 studies that sampled a total of almost 4,800,000 students, Szumski and colleagues (2017) found that the academic achievement of students without special learning needs in inclusive middle and high school classrooms is not negatively impacted. Notably, the lack of harm to general education students in inclusive classrooms held even when students with severe educational needs and emotional and behavioral disorders were present in the classroom (Szumski, Smogorzewska, & Karwowski, 2017).<sup>2</sup>

## Appropriateness of productive struggle

Teaching standards-based curriculum is complex and requires multiple modes of instruction. For example, mathematical discussions make mathematics explicit as students verbalize connections to prior content and collectively work to formalize mathematical concepts with canonical vocabulary (*5 Practices* book, Smith & Stein, 2011). This is especially important during lesson launch and closure as students with identified exceptionalities can struggle to get started and to put it all together.

There are several common pitfalls when trying to support students with identified exceptionalities to engage in standards-based curricula:

1. *Pitfall 1: Hinting*, which typically reduces the cognitive demand of the task and thereby removes the struggle and the learning;<sup>3</sup>
2. *Pitfall 2: Backgrounding Problem Context*, which leads to an overemphasis on procedures and strips mathematical problems of meaning;<sup>4</sup> and
3. *Pitfall 3: Providings Formulas*, which removes students' opportunity to engage in the actual mathematics content. Because these pitfalls remove productive struggle, they are (unfortunately) some of the core strategies in special education interventions.

(Lynch et al., 2018)

## Instructional strategies for broadening access

Scaffolds for participation in teamwork, problem-solving, and in engagement in whole-class mathematical discussions are necessary to promote equitable learning. See the appendix for culture-building teacher moves for supporting students with autism in standards-based mathematics (Table 1, Lambert et al., 2020) and teacher moves for supporting students with identified exceptionalities in problem-solving and mathematical discussions (Table 2, Lambert & Sugita,

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<sup>2</sup> Szumski et al. (2017) noted that while there was no difference in achievement in classrooms with and without students with severe educational needs and emotional and behavioral disorders, there was a statistical tendency for lower achievement of general education students in classroom with students with emotional and behavioral disorders.

<sup>3</sup> Hinting also creates algorithmic thinking and positions the diminishes students' mathematical authority. Hinting can also obfuscate students' opportunities to make connections across representations.

<sup>4</sup> Removing the mathematical context also reduces the need to have collaborative discussion. This strips the problems stripped of meaning and so also strips the learner of engagement and curiosity.

2016). Using these strategies, students who have “never offered more than one-word responses during whole group instruction” have been shown to shift participation by the end of the year to have “equal rates of engagement to their nondisabled peers” (Lambert & Sugita, 2016, p. 359).

In addition, Pfister and colleagues (2015) found that curricular materials were able to support teachers to engage in important scaffolds such as using manipulatives and helping students to focus on the important aspects of the lesson; however, more interactional “micro-scaffolds” such as stimulating discourse, cognitive activation (e.g., What do you notice? What did you have to do so that ...?), and handling errors productively were much harder for teachers because they required in-the-moment decisions responsive to student needs. Examples of what these five scaffolds look like when they are done well (and not) can be found in the rubric in Table 3 (see the Appendix).

## Strategies for starting word problems

Research in special education supports students with word problems through schema-based instruction, which involves unpacking the problem’s structure before the student explores the problem, in essence removing the inquiry (Browder et al, 2018; Jitendra et al., 2015). Recent modified versions of schema-based instruction are more complex but similarly inhibit opportunities for conceptual learning, for example by providing students with graphic organizers that are specific to a problem-type, explicit instructions, “rules taught as chants with hand motions representing the underlying problem structures,” and more (Browder et al, 2018). Altering tasks so that students must collaboratively create graphic organizers and other visual representations of the problem may accomplish the same purpose for students with identified exceptionalities as well as support the learning of students without specialized learning plans.

One exploratory study (Lambert & Sugita, 2016) found that restating word problems (while retaining the problem type) and re-reading word problems in small chunks that students then model step-by-step can mitigate language difficulties while maintaining cognitive demand (p. 358).

In a manual designed for teachers, Cole et al. (2000) suggest strategies such as making oral recordings of the text so that students can listen to and rewind the written material instead of reading it, providing students with the word problem text or recording in advance so that they can become familiar with the context of the problem, and discussing the problem’s context in teams (Alquraini & Gut, 2012).

Strategies from this section on supporting students with literacy such that they are able to begin problem-solving with word problems are summarized in the appendix in Table 4.

## Mitigating status issues

Because teamwork is a key component of standards-based curriculum and instruction, teachers must carefully attend to issues of status so that students with identified exceptionalities are not marginalized and stigmatized by their peers. For example, students can be excluded from making mathematical decisions during teamwork, instead being delegated to non mathematical features of the task such as material management (Baxter, Woodward, & Olson, 2001). Exclusion from meaningful work on the mathematical aspects of the task is likely to induce a negative stigma on students’ mathematical competence.

This exclusion can be mitigated by giving students explicit instruction on how to work together so that students with higher proficiency levels do not take-over the mathematical thinking for students with identified exceptionalities

(Bottge, Heinrichs, Mehta, & Hung, 2002; Cohen & Lotan, 2014; Horn, 2012). For a list of such instructional strategies, see Table 5 in the appendix.

Teachers must also become aware of any of their own biases, as teacher “attitudes and values not only influence the attitudes and values of [...] students, but they can affect the way you teach, particularly your assumptions about students [...] which can lead to unequal learning outcomes for those in your classes” (Davis, 2010, p. 58, quoted in Thurber & Bandy, 2018). For example, students with identified exceptionalities in inclusive general education classrooms are sometimes called on fewer times than other students (Bottge, Heinrichs, Mehta, & Hung, 2002). This may require teachers to collaborate with colleagues, such as in co-teaching situations.

## Co-teaching

While co-teaching is not possible in all contexts, many conjecture that co-teaching is a productive way to support equitable instruction for students with identified exceptionalities in inclusive classrooms.

According to Sileo and van Garderen (2010), co-teaching happens when general education and special education teachers work together in an inclusive classroom by collaborating on multiple dimensions of instruction, including planning, teaching, and assessing for all students, not just students who qualify for special services. Not only might such co-teaching help mitigate status issues, but it can also allow for coherent, integrated support for students with identified exceptionalities. Ideally, co-teaching fully leverages the strengths of special education teachers such as through team teaching rather than the common “one teach, one assist” model.

Importantly, co-teaching should not be forced, as teacher attitudes in co-teaching situations can impact the tone for classrooms and impact student learning (Rexroat-Frazier & Chamberlin, 2019, p. 178, citing Sakiz, Pape, and Hoy, 2012). Desirable attitudes include mutual caring, interest, concern, encouragement, and high expectations.

In addition, co-teachers should have relatively equal professional standing — co-teaching is not an opportunity for mentorship — and should negotiate their respective roles so that expectations for distributions of labor and responsibility are explicit (Rexroat-Frazier & Chamberlin, 2019, p. 175 citing Walther-Thomas, Bryant and Land, 1996).

Finally, co-teachers should be relatively aligned with their philosophy of education, meaning how they view the purpose of their profession (Rexroat-Frazier & Chamberlin, 2019 citing Magiera et al., 2005).

Because paraprofessionals are not on equal standing with teachers, this definition does not apply to paraprofessional aid. In classrooms fortunate enough to have paraprofessionals, it is important to ensure that their presence does not interfere with the classroom teacher’s sense of instructional responsibility for students with identified exceptionalities or that the paraprofessional’s proximity to students interfere with peer-to-peer relationships or foster dependence (Cook & Cook, 2020, p 144).

## Designing for inclusion

With or without a co-teacher, differentiation in inclusive classrooms can feel overwhelming. Instead of differentiating in the traditional sense, differentiation can be front-loaded in curricula through flexible designs that support multiple-ability engagement. This is called Universal Design for Learning, or UDL. According to Thurber & Bandy

(2018), UDL involves “flexible goals, methods, materials, and assessments ... rather than approaching accessibility as an afterthought or *only* on a case-by-case basis” (*emphasis added*). Examples of UDL include designing for opportunities for multi-modal engagement with mathematical concepts (e.g., algebra tiles) and for participation structures that distribute labor in ways that support students with identified exceptionalities to engage in cognitively demanding tasks (e.g., team roles).

In particular, UDL supports students to learn through collaboration. While collaboration is more challenging for some students with identified exceptionalities, it is an essential 21st century skill, not to mention that it can support students with identified exceptionalities’ learning through increasing opportunities for one-on-one support through the mentoring peers (CAST, 2018). Collaboration needs to be carefully structured in order to mitigate status issues, as previously mentioned, such as through sentence starters that support students to ask each other for help, team roles that foster multiple-ability participation, and teamwork norms and rubrics (CAST, 2018).

## Summary

So, how can students with identified exceptionalities be supported to experience productive struggle during collaborative problem-solving on cognitively-demanding tasks?

First, mindsets must shift from thinking about dis-ability as a kind of brokenness. This kind of deficit mindset results in instructional neglect of students’ existing capacity for learning, their special abilities, and their potential. As Susan Robinson — a distinguished alumni of Penn State University and CEO/founder of Global Health AspirAction, and a person with a genetic visual impairment — says in her Ted Talk, “The term ‘disability’ detonates a mindset of *less than*” (Robinson, 2016), a clear conflict with growth mindsets shown to be important for mathematical achievement (Boaler, 2015; Bostwick et al., 2017).

Second, students with identified exceptionalities can experience productive struggle during cognitively-demanding tasks when they are supported through ambitious instructional strategies, some of which are identified in Tables 1-3 in the Appendix.

Third, access to word problems can be increased with small modifications that may benefit all students, such as by recording a read-aloud of the problem that students can rewind and relisten to in their groups. More strategies can be found in Table 4.

Fourth, it is critical to attend to status issues in inclusive classrooms that use teamwork to engage students in cognitively demanding tasks. Table 5 provides multiple research-based strategies for mitigating status issues.

Fifth, co-teaching may support all student learning in inclusive classrooms by re-distributing the labor of teaching across two teachers with differing, complementary expertise. Research indicates that co-teaching may be most productive when both teachers are willing and eager to work together to plan, teach, and assess; are equals in professional standing; have clearly defined each teacher’s responsibilities in the classroom; and similarly orient to the purpose of teaching (i.e., their philosophy of education).

Finally, mathematical tasks should be designed to have multiple entry points and engage students through multiple modalities with their peers in cognitively-demanding problem-solving tasks. This is called Universal Design for Learning. Some argue that UDL is not radical enough and that designs for learning (and, in general) should design for *disability first* as a way to benefit all learners. This flips the traditional approach to designing for the imaginary “average”

learner upside down, and it may have promising results. A proponent of this approach, Elise Roy, has given lectures on this design stance at leading design firms such as Microsoft, NASA, AIGA, and the U.S. Institute for Peace (Roy, 2015). Surely if innovators such as NASA see value in designs that re-able those who have previously disabled, there may be value in exploring disability-first designs for learning as well.

By maintaining asset-based perspectives of and high-expectations for students with identified exceptionalities, including severe exceptionalities, we can expunge barriers to conceptual learning and foster scaffolds for meaningful engagement for all students.

## References

See the full report for a list of citations.



## Appendix

Table 1.

*Instructional strategies for supporting students with autism in inclusive standards-based classrooms* (Lambert et al., 2020, p. 508-509)

<b>Supports for students with autism to participate in standards-based classrooms</b>	
Begin with relationships	Establish strong relationships from the beginning of the year, especially through finding shared interests.
Strengths-based views of exceptionality	Verbally notice both mathematical and social strengths when talking to and about students. Do so using non-medical language (e.g., “shy” instead of “nonverbal”). Ask questions to elicit thinking and then help students build from their current understandings. Pay attention to specifically what is challenging for students, such as verbal participation. Consider asking students’ permission before sharing their thinking in front of a team or the whole class.
Make norms of mathematical discussion very explicit	<p>Have the class define and describe what discussion looks and sounds like and create a durable, visible record of this discussion. For example, have the class work collectively to create a chart with an eye on one side (“looks like”) and an ear on the other (“sounds like”) with each side filled in appropriately. Students may generate ideas such as:</p> <p><i>Mathematical discussions are:</i> when you talk about math and what it can do, talking about how we use strategies, when two or more people have different answers, sharing ideas with others</p> <p><i>Looks like:</i> Notebooks out, eyes on the speaker, showing work to each other, taking notes on what other people are saying, agree on an “agree symbol”</p> <p><i>Sounds like:</i> “I agree with you because...,” “I respectfully disagree with you because...,” “This is how I did my work,” “I don’t understand the strategy,” “Can you repeat that?,” “I’d like to add on to what you said,” “My strategy has a connection with yours,” and “Can you explain more?”</p> <p>(p. 505, direct quotation from a student conversation poster image)</p>
Scaffolded discussion with peers	Intervene in teamwork to support students to share out. Direct students to work with specific peers and physically move their notebooks or papers to be next to each other, then check in on their progress. Hold peers accountable even if students are quiet talkers and thus hard to hear.
Collaborative shares	Have students share out in pairs so that students who do not prefer verbal interaction can still participate.
Notice students’ participation	Some students may participate differently than others. For example, instead of raising their hand high, a student may raise just one finger slightly. Be sure to notice and respond to such participation as quickly as possible.

Table 2.

**Problem-solving supports for students with identified exceptionalities**

Multi-modal curriculum design	Provide students with choices about what materials are used to solve problems (equations, drawings, algebra tiles, connecting cubes, base-ten blocks, etc.)
Consistent routine	Teacher-led lesson launch, individual or team problem-solving, and then a whole class discussion in which students present their strategies and solutions
Teacher scaffolds for problem solving	Scaffold the starting problem-solving by restating word problems (while retaining the problem type) and re-reading word problems in small chunks that students model step-by-step. This reduces students' difficulties with language but does not reduce the cognitive demand of the mathematics task.
Equitable teamwork	Mitigate marginalization by providing additional support to small groups (this often requires teacher professional development)

**Mathematical discussion supports for students with identified exceptionalities**

Student rehearsal of strategy shares	Allow students to rehearse the strategy they will share out in discussion by providing them with a paraprofessional
Access to manipulatives and notebooks	Allow students to use their notebooks as a record of their problem-solving for as long as they need in order to support their participation in discussion; allow students to use manipulatives such as algebra tiles rather than equations to model their mathematical thinking during discussions
Teacher questioning	Hold students accountable for explanations of their strategies by asking multiple follow-up questions

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Table 3.

*Rubric for scaffolds.* Examples from Pfister, Opitz, & Pauli (2015).

Scaffold Definitions		Teacher Actions Rubric		
Scaffolds and example scaffolding questions	Goals	(0)	(1)	(2)
<p><b>Cognitive Activation</b></p> <p>Compare!</p> <p>What do you notice?</p> <p>What did you have to do so that ...?</p>	<p>Poses clear, content-related, meaningful, challenging questions and problems, provides stimulation for describing or substantiating facts, observations, etc.</p> <p>Enables the establishment of relationships between contents</p> <p>Stimulating discourse</p>	<p>Set tasks with small steps</p> <p>Told the students which actions they have to carry out</p> <p>Posed questions which require a one-number answer</p> <p>Carried out actions with the manipulatives himself</p>	<p>Carried out actions with the manipulatives himself</p> <p>Often told students solution steps</p> <p>Sometimes requested observation, description, or substantiation of facts and findings</p> <p>Sometimes requested comparison of solution strategies</p>	<p>Constantly requested students to verbalise and substantiate their solution steps</p> <p>Allowed problems (even correctly solved ones) to be discussed</p> <p>Invited the formulation of insights and observations</p>
<p><b>Stimulating Discourse</b></p> <p>Describe what you have done!</p> <p>Can you explain that in more detail?</p> <p>Are there any other ways how we can solve it?</p> <p>Can we write/calculate it differently?</p>	<p>Invites the students to comment on contributions or actions of others</p> <p>Responds to students' contributions</p> <p>Initiates reflections on solution strategies</p>	<p>Asked for numbers, results</p> <p>Let the students finish sentences he has started</p> <p>Spoke most of the time</p>	<p>Formulated central findings himself</p> <p>Primarily asked for results or subsequent steps</p> <p>Let the students finish sentences he has started</p> <p>Sometimes let the students "dictate" the next steps</p> <p>In isolated cases, he incorporated contributions from the</p>	<p>Asked for reflections</p> <p>Let thought processes and insights to be presented</p> <p>Mostly does not interrupted the students' contributions</p>

			children into the class discussion	
<p><b>Handling Errors Productively</b></p> <p>Where are you stuck?</p> <p>What are you considering?</p> <p>How did you find it out?</p> <p>How can we find out whether that's correct?</p>	<p>Recognises the learning potential or difficulty of a situation</p> <p>Intervenes in the learning processes in a supportive manner</p> <p>Endeavours to understand the students' solution strategies or reflections</p> <p>Supports students in tackling problems independently</p> <p>Checks the students' understanding following the intervention</p>	<p>Demanded that certain procedures to be carried out</p> <p>When students were uncertain, he told them how to continue</p> <p>Rubbed out mistakes and wrote down the solution himself</p> <p>Pointed to what was written on the blackboard</p>	<p>Provided hints for using the structure of the Dienes blocks</p> <p>Requested the students to try the problem again with help of the manipulatives</p> <p>Demanded more careful work (not specifically mathematical)</p>	<p>Requested verbalisation of the procedure</p> <p>Requested substantiation and proof</p> <p>Provided feedback on systematic procedures</p> <p>Let insights from a mistake to be explicitly formulated, or the mistake to be "named"</p> <p>Established connections with other solved problems or problems that have not yet been solved</p>
<p><b>Target Orientation</b></p> <p>Describe the rule/pattern!</p> <p>Why does it have to be done like that?</p>	<p>Focuses on core content elements</p> <p>Demonstrates what is important, points out conventions</p> <p>Summarises important findings, recapitulates these findings in his/her own words</p>	<p>Focused on carrying out the procedure correctly</p>	<p>Formulated central findings</p> <p>Always pointed out important things</p> <p>Recapitulated insights or relevant things</p>	<p>Summarised the students' thoughts</p> <p>"Translated" student contributions</p> <p>Let insights to be formulated and summarised</p> <p>Worked out key characteristics and procedures</p>

<p><b>Using Manipulatives</b></p> <p>Can you show that with manipulatives?</p> <p>Can we place/do it differently</p>	<p>Employs manipulatives to support the learning process .76 (.90)</p> <p>Allows facts to be represented actively using manipulatives</p> <p>Emphasises the understanding of structure or the systematic use of the manipulatives</p>	<p>Let students name the units of the Dienes blocks</p> <p>Mostly manipulated the Dienes blocks himself</p> <p>Told the students what they should do with the Dienes blocks</p> <p>Mostly wrote down the problem solution by himself</p>	<p>Encouraged students to use the Dienes blocks in a structured manner</p> <p>Let the structure of the Dienes blocks be used clearly for the grouping or de-grouping process and for recording (interim) results</p> <p>In part, he established the relationship between manipulatives, representations and notations</p> <p>Addressed the difference between an empty number line and a number line</p>	<p>Let notation forms and arithmetic steps to be compared</p> <p>Worked out the characteristics or differences of the manipulatives, representations and notations clearly on several occasions (e.g. difference between an empty number line vs. and a number line)</p> <p>Let different presentation forms to be used for individual solution strategies</p>
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Table 4.

*Instructional strategies for supporting students with literacy in word problems such that they can begin mathematical problem-solving tasks.*

<b>Strategies for helping students get started on word problems</b>
Restate word problems (while retaining the problem type) (Lambert & Sugita, 2016)
Re-read word problems in small chunks that students model step-by-step (Lambert & Sugita, 2016)
Make oral recordings of the text so that students can listen to and rewind the written material instead of reading it (Cole et al., 2000 as cited in Alquraini & Gut, 2012)
Provide students with the word problem text or recording in advance so that they can become familiar with the context of the problem (Cole et al., 2000 as cited in Alquraini & Gut, 2012)
Discuss the problem context in small groups (Cole et al., 2000 as cited in Alquraini & Gut, 2012)
Require teams to collaboratively create graphic organizers and other visual representations of the problem (modified from Browder et al., 2018)

Table 5.

*Instructional strategies for cultivating equal-status interactions in teamwork — and “equity pedagogy”* (Bannister, 2016)

<b>Strategies for cultivating equal-status interactions in teamwork</b>
<b>Multiple-ability strategy</b>
Create a classroom culture that values mistakes and rough-draft thinking (Nasir, Cabana, Shreve, Woodbury and Louie 2014)
Use accountability structures that hold each team member accountable for the group’s shared work (Nasir et al., 2014)
Use random assignment of team roles (Nasir et al., 2014)
Press all students for high levels of justification (Nasir et al., 2014)
Share clear evaluation criteria with students (Nasir et al., 2014)
Use “groupworthy” tasks (Cabana, Shreve, and Woodbury, 2014; Cohen & Lotan, 2014) by: <ol style="list-style-type: none"> <li>1. focusing on the big ideas of a lesson</li> <li>2. providing tasks that afford multiple solution pathways and/or require multiple representations</li> <li>3. providing tasks that require multiple intellectual abilities — finding information, problem-solving, basic skills, or material organization — such that no single individual can possess all of them</li> </ol>
List out the intellectual abilities the task requires to students and then say something like, “None of us has all of these abilities that are required for this task. Everyone has some of these abilities, and so everyone will have something important to contribute to our shared work today. Listen carefully to one another, as you will all be important resources for your group.” (Bannister, 2016, p. 342)
<b>Assigning competence strategy</b>
Make public, positive, evaluative statements that recognize specific intellectual contributions that students with identified exceptionalities make during team work (Horn, 2012). This can be done for other low-status students as well, such as students who are marginalized along lines of gender, race, social class, physical attractiveness, and prior academic performance (Bannister, 2014; Cohen & Lotan, 2014).