Rough-Draft Talk in Mathematics Classrooms

Amanda Jansen, Brandy Cooper, Stefanie Vascellaro, and Philip Wandless

Tricia (*a seventh grader*)*:* I'm kind of really shy, so I'm, like, super conscious about when it comes to answering in front of people. . . . I was, like, always nervous that it would be wrong (Jansen 2006, p. 416).

Some students are reluctant to participate in whole-class discussions. But if they do not participate, their peers will not learn from them. During discussions, many students believe that they should perform a "final draft" of correct and complete solutions (Jansen 2009). How can we create a classroom culture that supports continued learning during classroom discourse?

Exploratory (or rough-draft) talk is one such productive strategy. Learning anything new involves "working on understanding" (Barnes 2008, p. 3). Rough-draft talk looks like false starts, expressions of uncertainty, and incomplete or imperfect sentences.

Rough-draft talk is talking to learn. Familiar to students from language arts, rough-draft talk occurs when ideas become more connected and organized. Similar to writing, rough-draft talk is an iterative process.

Creating a normative practice of rough-draft talk supports the engagement of more students. If rough-draft talk is valued, brainstormed ideas are welcomed. More students are likely to take risks rather than freeze during challenging tasks. Valuing a wider range of contributions invites greater involvement, in contrast to the same students who participate frequently or not at all.

When teachers create spaces for rough-draft talk, they continue to choose mathematical tasks that promote particular understandings, but they adjust their classroom discourse practices. For instance, to promote rough-draft talk, teachers talk more explicitly about how people learn and the role of talk in learning, highlighting that learning takes time and that talking through in-progress ideas supports learning. Three principles and practices support rough-draft talk. (See **table 1**.)

Principle 1: Foster a culture supportive of intellectual risk taking. Explicitly tagging initial discussions of solutions as "rough drafts" encourages students to share in-progress thinking. This tagging reduces the threat of being wrong. A nonevaluative stance by the teacher empowers students.

To create a culture of risk taking, a teacher used a nonevaluative routine to discuss students' thinking. She displayed a task and directed students to first use rough-draft talk in small groups. Groups shared initial ideas

Edited by **Alessandra King**, alessandra.king@holton-arms.edu, Holton-Arms School, Bethesda, Maryland, and **Julie Amador**, jamador@uidaho.edu, University of Idaho, Coeur d'Alene. Readers are encouraged to submit manuscripts through **http://mtms.msubmit.net**.

(rough-draft presentations) at a document camera, and the solutions were not evaluated. Students asked questions of their peers only when they needed clarification.

Next, students returned to their groups and revised their thinking toward final-draft solutions, comparing their original ideas to presenters' ideas. During a second round of presentations (final drafts), students were encouraged to critique their peers' approaches. Then the teacher asked questions to guide students' thinking while highlighting important ideas.

The teacher posed the task in **figure 1** to her sixth graders. She *expected* students to struggle with deciding on the correct operation for the problem: multiplication or division. When directing students to work in small groups, the teacher said, "This is just your rough-draft thinking right now." Students drafted solutions on dry-erase boards.

Before students shared rough-draft thinking at the document camera, the teacher said, "Remember, this is just our rough draft, so we're not going to say if they're right or wrong. But we might want to ask some questions."

When members of two groups illustrated finding an answer of 10 by multiplying 5/6 by 12, a peer asked, "Why did you choose to multiply?"

The presenter responded, "I feel like it would be a quicker way."

Another group's rough-draft thinking involved converting 5/6 of a foot and 12 feet to inches. While showing the calculations at the document camera, a student explained, "We took the 12 and did 12 times 12 and got 144. And then we did 2 times 5, which is 10, and then did 10 divided into 144 and got 14 pieces."

A student asked, "Why did you divide 10 into 144? How come you did not multiply 144 and 10?"

The presenter explained, "We would get too high of a number if we did that [multiplied 10 times 144]."

Table 1 These three principles and practices support rough-draft talk.

Principles	Practices
1. Foster a culture supportive of intellectual risk taking.	Tag talk as rough drafts. Engage in nonevaluative sharing prior to critique.
2. Promote the belief that learning mathematics involves revising understanding over time.	Provide opportunities for students to revise their thinking.
3. Raise students' statuses by expanding on what counts as a valuable contribution.	Strategically call on students with helpful in-progress ideas and position them as competent mathematics students. Explicitly identify instances of in-progress ideas that helped the class move forward in its understanding.

Fig. 1 This division of fractions task was designed to have a remainder, with the expectation that sixth-grade students would struggle with deciding which operations to use.

Lucy is making bracelets using ribbon. She has a ribbon that is 12 feet long. Each bracelet needs a piece of ribbon that is 5/6 of a foot long. How many pieces will she be able to cut?

Fig. 2 This task promoted revision.

My Favorite Rough Draft

Directions: Read and answer the problem with your shoulder partner. Explain whether the expression on the left side of the equal sign is equivalent to the expression on the right side. Show all work.

$$14x^2 + 6 - 9x^2 + 4x = 9x^2 + 6$$

The tag of "rough-draft" thinking promoted an open space for exploring initial solutions. A second round of group work followed. During final draft discussions, all groups decided that division was the appropriate operation and that the answer should be 14 instead of 10.

Principle 2: Promote the belief that learning mathematics involves revising understanding over time. Revising mathematical thinking promotes learning through refining ideas. To enact the routine of "My Favorite Rough Draft," the class revised a student's explanation. ("My Favorite Rough Draft" is a modification of "My Favorite No," https://www.teachingchannel.org/videos/class-warm-up-routine, in which a teacher analyzes a student's error.) During revision, the class affirmed what was initially productive about the explanation.

Students received a task on paper that had been inserted into a plastic sheet cover (see **fig. 2**). They wrote initial explanations in pairs and then used overhead projector pens to write

on nonpermanent surfaces. This process promotes exploratory work (Liljedahl 2016).

This teacher wanted students to analyze the error in **figure 2**, but she wanted students to go beyond catching the error to elaborate on meanings underlying procedures. She also wanted to promote mathematically precise language, such as "combine like terms," and the use of vocabulary, such as *coefficient*, *constant*, *variable*, *expression*, and *exponent*. Students' initial explanations were assumed to be in rough-draft form. The students discussed initial explanations with a partner. An example of such rough-draft talk follows:

Student: But then you have to think about it. You can't do an exponent if you're using . . . so, it's $5x$ to the second power. You can't do $5x$ to the second power plus $4x$. It doesn't work. Because $4x$, it would have to go to the second power. You can't add a second power to that.

After writing a first draft, students received peer feedback, in which two pairs exchanged explanations. Next, the class revised one explanation: the teacher's favorite (anonymous) rough draft. All together, they discussed how to revise toward a final draft using more precise language.

Principle 3: Raise students' statuses by expanding on what counts as a valuable contribution.

When rough-draft talk is recognized as valuable for supporting learning, then more students can be positioned as competent mathematical thinkers (Cohen and Lotan 2014; Featherstone et al. 2011; Horn 2012). A teacher can ask a student to share, even if he or she is struggling to understand. In so doing, the teacher is making a public statement that this rough-draft talk is useful for the class's learning.

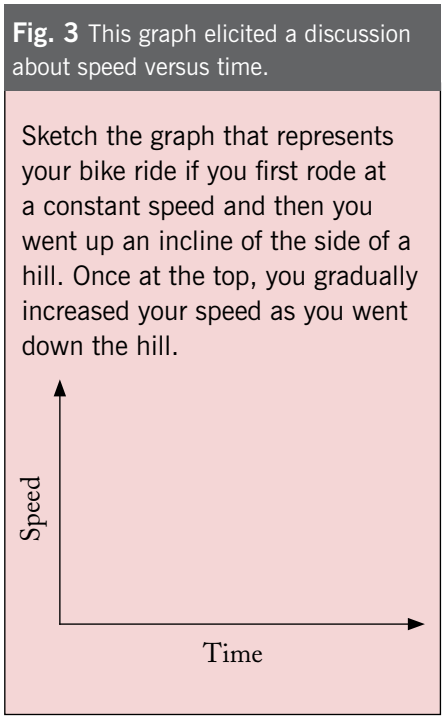
An opportunity to raise a student's status occurred when a class discussed qualitative graphs about speed depending on time. The teacher conjectured that when students first graph this situation, they would graph elevation versus time. (See **fig. 3**.) Students might draw an uphill line to initially represent pedaling uphill (rather than a downhill line to represent speed decreasing over time). Qualitative graphs can be used informally to get students to explore motion and recognize a change in rate at points.

A student shared her graph with the class. The teacher noticed that Jakeel wondered about it, so she called on Jakeel to share his rough-draft talk.

Jakeel: About the part that's going down. How can you write that when you're already at the top of the hill? You're on your way up, but the straight line is above the top of the hill. You can't start on the top of the hill if you're going up.
Teacher: Can anyone answer that? It's a good question.
Jakeel: The part that's going down is at the top of the hill. How can you be above the top of the hill? That means you'd have to be in the air.
Teacher: Come show us the points that you're talking about.
Jakeel: [Jakeel is at the overhead projector and points to the lowest point on the graph.] Right here. If that's the top of the hill, how can it be up here [pointing to the straight line for constant speed]? It would be in the air. [See **fig. 4**.]
Student: Because the y-axis is the speed. Not the hill [elevation], but the speed.

After continued discussion, the teacher explicitly pointed out how

Fig. 3 This graph elicited a discussion about speed versus time.



Jakeel's question allowed the class to have a better understanding of speed versus time.

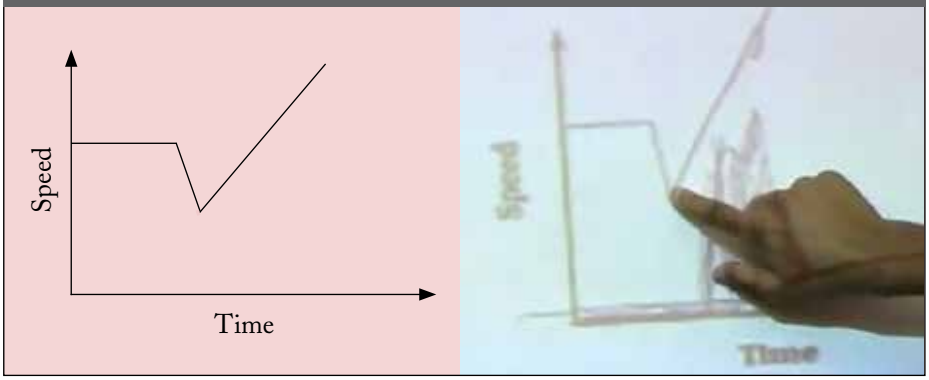
Teacher: How many people thought similarly to Jakeel at first? [Students raise hands.] See, you're not alone. It helped everyone to talk about this and understand it.

Jakeel's contribution was publicly treated as helpful and productive for the class. His rough-draft talk helped other students clarify their thinking.

ACROSS THE CLASSROOMS

Explicit promotion of rough-draft talk takes productive classroom discourse to a higher level because it provides a safe space for students to develop understanding. When teachers developed new routines for nonevaluative sharing and promoted public revisions, students' in-progress thinking was positioned as valuable. Such actions went going beyond the classrooms' typical discourse. When these teachers asked their students how they felt about being incorrect in front of their peers, they made comments like these:

Fig. 4 Jakeel's confusion was alleviated when a peer explained that the *y*-axis represented the speed, not the hill elevation.



- "I like it even when I'm wrong, because I learn from my group and get better."
- "It was a mistake that everyone makes. No big deal. Just fix it. Everyone will know that you might need help, and they will support you and teach you."

Engaging students in rough-draft talk promotes learning because whole-class discussion can foster understanding.

REFERENCES

Barnes, Douglas. 2008. "Exploratory Talk for Learning." In *Exploring Talk in School*, edited by Neil Mercer and Steve Hodgkinson, pp. 1–15. London, UK: Sage Publications.

Cohen, Elizabeth G., and Rachel A. Lotan. 2014. *Designing Groupwork: Strategies for the Heterogeneous Classroom*. 3rd ed. New York: Teachers College Press.

Featherstone, Helen, Sandra Crespo, Lisa M. Jilk, Joy A. Oslund, Amy Noelle Parks, and Marcy B. Wood. 2011. *Smarter Together! Collaboration and Equity in the Elementary Math Classroom*. Reston, VA: National Council of Teachers of Mathematics.

Jansen, Amanda. 2006. "Seventh Graders' Motivations for Participating in Two Discussion-Oriented Mathematics Classrooms." *Elementary School Journal* 106 (May): 409–28.

———. 2009. "Prospective Elementary Teachers' Motivation to Participate in Whole-Class Discussions during Mathematics Content Courses for Teachers." *Educational Studies in Mathematics* 71 (June): 145–60.

Horn, Ilana S. 2012. *Strength in Numbers: Collaborative Learning in Secondary Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Liljedahl, Peter. 2016. "Building Thinking Classrooms: Conditions for Problem Solving." In *Posing and Solving Mathematical Problems: Advances and New Perspectives*, edited by Patricio Felmer, Erkki Pekhonen, and Jeremy Kilpatrick, pp. 361–86. New York: Springer.

TeachingChannel. "My Favorite No: Learning From Mistakes." https://www.teachingchannel.org/videos/class-warm-up-routine




Amanda Jansen, jansen@udel.edu, teaches elementary school and middle school prospective mathematics teachers in the School of Education at the University of Delaware in Newark. **Brandy Cooper**, bcooper@msd.k12.de.us, teaches sixth-grade mathematics at Milford Central Academy in Milford, Delaware. **Stefanie Vascellaro**, stefanie.vascellaro@appo.k12.de.us, is a sixth-grade mathematics teacher at Everett Meredith Middle School in the Appoquinimink School District in Middletown, Delaware. **Philip Wandless**, philip.wandless@smyrna.k12.de.us, is an eighth-grade mathematics teacher at Smyrna Middle School in Smyrna, Delaware. All four authors are invested in improving classroom discourse to engage more students in learning mathematics.