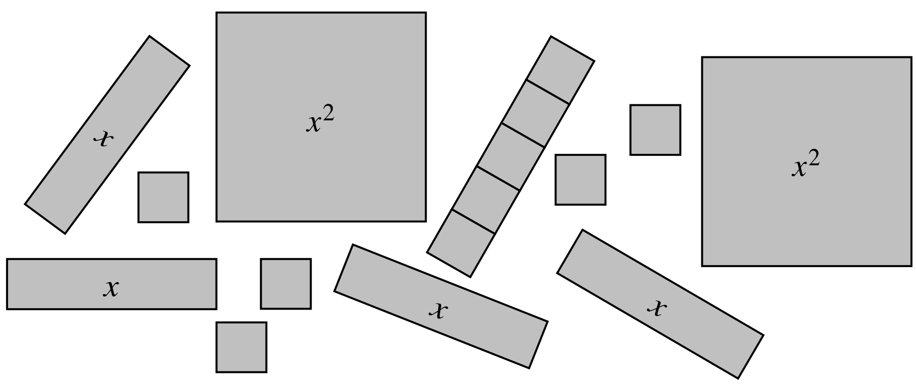
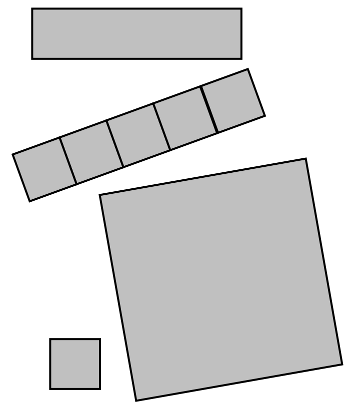
Using Appropriate Tools Strategically: Algebra Tiles are Not Just for Factoring— Single Variable Algebraic Work



Area of a Rectangular Shape

In this lesson, you will continue to study variables in more depth by using them to describe the dimensions and areas of different shapes. Then you will organize those descriptions into algebraic expressions.

1. AREAS OF ALGEBRA TILES

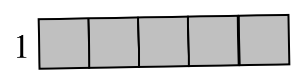


Your teacher will provide your team with a set of algebra tiles. Remove one of each shape from the bag and put it on your desk. Trace around each shape on your paper. Look at the different sides of the shapes.

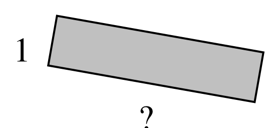
a. With your team, discuss which shapes have the same side lengths and which ones have different side lengths. Be prepared to share your ideas with the class. On your traced drawings, color-code lengths that are the same.

b. Each type of tile is named for its area. In this course, the smallest square will have a side length of 1 unit, so its area is 1 square unit. Thus, this tile will be called "one" or the "unit tile." Can you use the unit tile to find the side lengths of the other rectangles? Why or why not?

c. If the side lengths of a tile can be measured exactly, then the area of the tile can be calculated by multiplying these two lengths together. The area is measured in square units. For example, the tile at right measures 1 unit by 5 units, so it has an area of 5 square units.



The next tile at right has one side length that is exactly one unit long. If you cannot give a numerical value to the other side length, what can you call it?

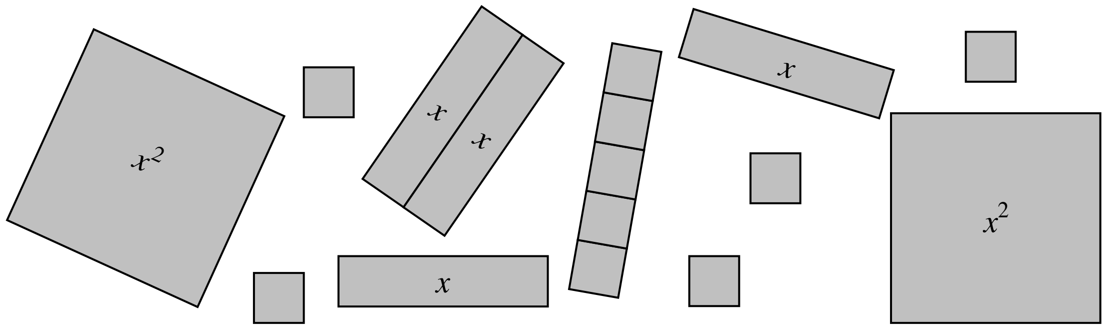


d. If the unknown length is called "x," label the side lengths of each of the four algebra tiles you traced. Find each area and use it to name each tile. Be sure to include the name of the type of units it represents.

https://technology.cpm.org/general/tiles/

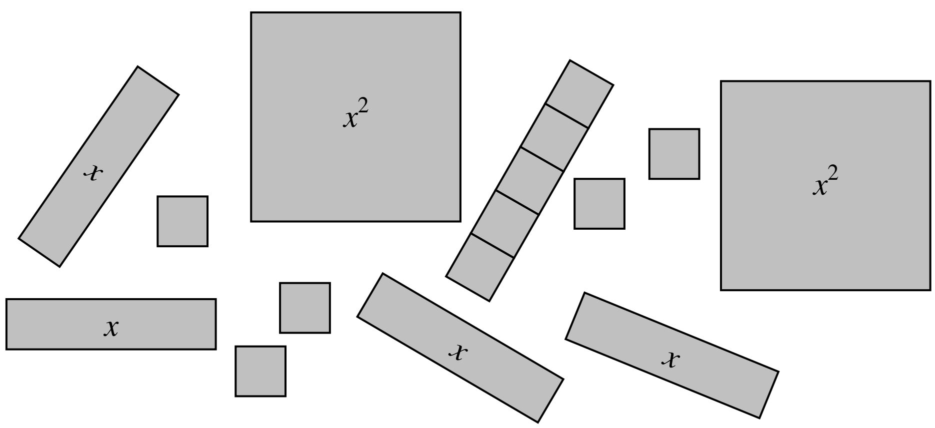
2. When a collection of algebra tiles is described with mathematical symbols, it is called an algebraic expression. Take out the tiles shown in the picture below and put them on your table.

- Use mathematical symbols (numbers, variables, and operations) to record the area of this collection of tiles.
- Write at least three different algebraic expressions that represent the area of this tile collection.

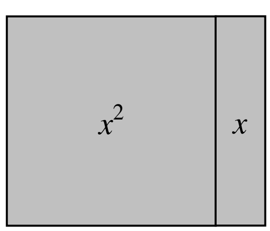


3. Take out the tiles pictured in each collection below, put them on your table, and work with your team to find the area as you did in problem 2.

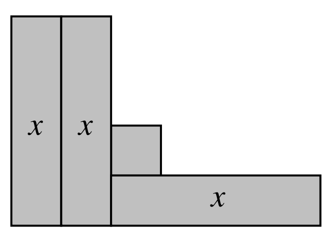
a.



b.



c.

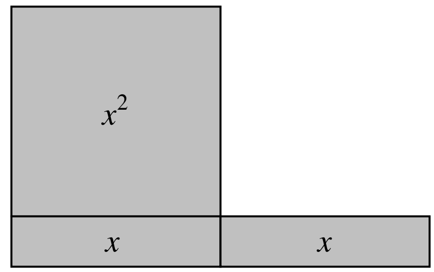


d. Is the area you found in part (a) the same or different from the area of the collection in problem 2? Justify your answer using words, pictures, or numbers.

Combining Like Terms

In the previous lesson, you used variables to name lengths that could not be precisely measured. Using variables allows you to work with lengths that you do not know exactly. Now you will work with your team to write expressions to represent the perimeters of different shapes using variables.

4. For the shape at right, one way to write the perimeter would be to include each side length in the sum: $x + x + x + 1 + x + x + 1 + x$.



   a. How many x lengths are represented in this expression? How many unit lengths?

   b. The expression above can be rearranged to $x + x + x + x + x + x + 1 + 1$ and then be written as $x(1 + 1 + 1 + 1 + 1 + 1) + 2$, which then equals $6x + 2$. Identify which property is used to rewrite the expression with parentheses.

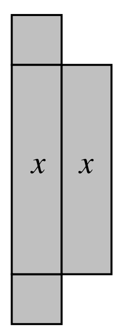
   c. **Like terms** are terms that contain the same variable (as long as the variable(s) are raised to the same power). **Combining like terms** is a way of simplifying an expression. Rewriting the perimeter of the shape above as $6x + 2$ combines the separate x-terms to get $6x$ and combines the units in the term to get 2.

   If you have not already done so, combine like terms for the perimeter of each of the different algebra tiles in problem 7.

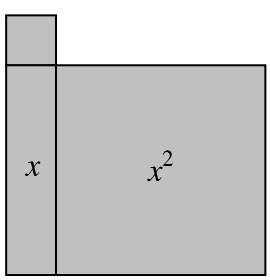
5. On your desk, use algebra tiles to make the shapes shown below. Trace each shape and label the length of each side on your drawing. With your team, find and record the total perimeter and area of each shape. If possible, write the perimeter in more than one way.

   a.

   b.



   c.



Evaluating Algebraic Expressions

In the previous lessons, you learned how to find the perimeter and area of a shape using algebra tiles. Now, you will challenge the class to find the perimeters and areas of shapes that you create.

6. In problem 5, x is a variable that represents a number of units of length. The value of x determines the size of the perimeter and area of the shape.

   Using the shapes from parts (b) and (c) in problem 5, sketch and label each shape with the new lengths given below. Rewrite the expressions with numbers and simplify them to determine the perimeter and area of each shape.

   a. $x = 6$

   b. $x = 2$

   c. Compare your method for finding perimeter and area with the method your teammates used. Is your method the same as your teammates' methods? If so, is there a different way to find the perimeter and area? Explain the different methods.

7. SHAPE CHALLENGE

You and your team will choose four algebra tiles. Then you will use them to build a shape to challenge your classmates. You may choose whatever tiles you would like to use as long as you use exactly four tiles.

As a team, decide on the shape you want to make. Experiment with different shapes until you find one you think will have a challenging perimeter and area for your classmates to find. Then, to share your challenge with the class:

- Build the shape with algebra tiles in the middle of your team so everyone in your team can see it.
- Get an index card from your teacher. On one side, neatly draw the shape and label each side.
- Write simplified expressions for the perimeter and the area on the same side of the card. This will be the answer key. Show all of your steps clearly.
- Turn the card face down so the answer is hidden. Then put the names of your team members on the top of the card. Place the card beside the shape you built with your algebra tiles.

Remember that your work needs to be clear enough for your classmates to understand.

Follow your teacher's directions to complete challenges created by other teams. As you look at their shapes, sketch them on your paper. Work with your team to label the sides and find the perimeter and area of each shape. Be sure to combine like terms to make the expressions as simple as possible.

8. Choose two of the shapes from problem 7. Sketch each shape and label it with its perimeter and area. Do not forget the correct units. It is not necessary to draw the figures to scale. Rewrite each expression with the values given below and then evaluate it.

a. $x = 1.5$ units

b. $x = 3\frac{3}{4}$ units

Inverse Operations

Variables are useful tools for representing unknown numbers. In some situations, a variable represents a specific number, such as the hop of a frog. In other situations, a variable represents a collection of possible values, like the side lengths of a picture frame. In previous chapters, you have also used variables to describe patterns in scientific rules and to write lengths in perimeter expressions. In this section, you will continue your work with variables and explore new ways to use them to represent unknown quantities in word problems.

9. THE MATHEMATICAL MAGIC TRICK

Have you ever seen a magician perform a seemingly impossible feat and wondered how the trick works? Follow the steps below to participate in a math magic trick.

Pick a number and write it down.
Add five to it.
Double the result.
Subtract four.
Divide by two.
Subtract your original number.
What did you get?

a. Check with others in your study team and compare answers. What was the result?

b. Does this trick seem to work no matter what number you pick? Have each member of your team test it with a different number. Consider numbers that you think might lead to different answers, including zero, fractions, and decimals. Keep track in the table below. For your convenience, a copy of this table is on the Mathematical Magic Tricks Resource Page.

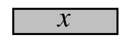
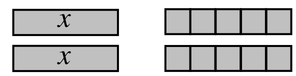
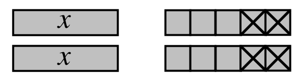
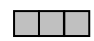
Steps	Trial 1	Trial 2	Trial 3
1. Pick a number.			
2. Add 5.			
3. Double it.			
4. Subtract 4.			
5. Divide by 2.			
6. Subtract the original number.			

c. Which steps made the number you chose increase? When did the number decrease? What connections do you see between the steps in which the number increased and the steps in which the number decreased?

d. Consider how this trick could be represented with math symbols. To get started, think about different ways to represent just the first step, “Pick a number.”

10. Now you get to explore why the magic trick from problem 9 works.

Shakar decided to represent the steps with algebra tiles. Since he could start the trick with any number, he let an x-tile represent the "Pick a number" step. With your team, analyze his work with the tiles. Then answer the questions below.

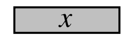
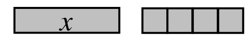
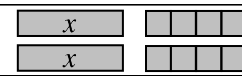
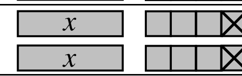
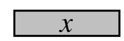
Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number.				
2. Add 5.				
3. Double it.				
4. Subtract 4.				
5. Divide by 2.				
6. Subtract the original number.				

a. For the second step, "Add 5," what did Shakar do with the tiles?

b. What did Shakar do with his tiles to "Double it"? Explain why that works.

c. How can you tell from Shakar's table that this trick will always end with 3? Explain why the original number does not matter.

11. Now reverse your thinking to figure out a new "magic trick." Locate the table below on the Mathematical Magic Tricks Resource Page and complete parts (a) through (c) that follow.

Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number.				
2.				
3.				
4.				
5.				
6.				

a. Use words to fill in the steps of the trick like those in the previous tables.

b. Use your own numbers in the trials, again considering fractions, decimals, and zero. What do you notice about the result?

c. Why does this result occur? Use the algebra tiles to help explain this result.

12. In the previous math "magic tricks," did you notice how *multiplication* by a number was later followed by *division* by the same number? These are known as **inverse operations** (operations that "undo" each other).

a. What is the inverse operation for addition?

b. What is the inverse operation for multiplication?

c. What is the inverse operation for "Divide by 2"?

d. What is the inverse operation for "Subtract 9"?

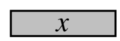
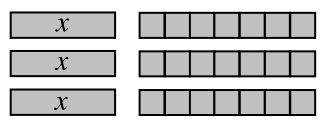
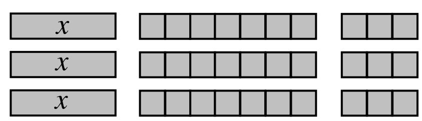
 MORE MATH FOR MORE PEOPLE

Distributive Property

In the previous lesson, you looked at how mathematical “magic tricks” work by using inverse operations. In this lesson, you will connect algebra tile pictures to algebraic expressions. An algebraic expression is another way to represent a mathematical situation.

13. In this problem you will consider a more complex math magic trick. The table you use to record your steps will have only two trials, but it will add a new column to represent the algebra tiles with an algebraic expression. To begin this activity, get a Distributive Property Magic Trick Resource Page from your teacher. Then:

- Work with your team to choose different numbers for the trials.
- Decide how to write algebraic expressions that represent what is happening in each step.

Steps	Trial 1	Trial 2	Algebra Tile Picture	Algebraic Expression
1. Pick a number.				
2. Add 7.				
3. Triple the result.				
4. Add 9.				
5. Divide by 3.				
6. Subtract the original number.				

14. For this number trick, the steps and trials are left for you to complete by using the algebraic expressions. To start, copy the table below on your paper and build each step with algebra tiles.

Steps	Trial 1	Trial 2	Algebraic Expression
1.			x
2.			$x + 4$
3.			$2(x + 4)$
4.			$2x + 20$
5.			$x + 10$
6.			10

a. Describe Steps 1, 2, and 3 in words.

b. Look at the algebra tiles you used to build Step 3. Write a different expression to represent those tiles.

c. What tiles do you have to add to build Step 4? Complete Steps 4, 5, and 6 in the chart.

d. Complete two trials and record them in the chart.

15. In Step 3 of the last magic trick (problem 14), you rewrote the expression $2(x + 4)$ as $2x + 8$. Can all expressions like $2(x + 4)$ be rewritten without parentheses? For example, can $3(x + 5)$ be rewritten without parentheses? Build $3(x + 5)$ with tiles and write another expression to represent it. Does this seem to work for all expressions?

16. You have been writing expressions in different ways to mean the same thing. The way you write an expression depends on whether you see tiles grouped by rows (like four sets of $x + 3$) or whether you see separate groups (like $4x$ and 12). The **Distributive Property** is the formal name for linking these two equivalent expressions.

Write each of the following descriptions in another way. For example, $4(x + 3)$ can also be written $4x + 12$. (Hint: Divide each expression into as many equal groups as possible.)

a. $6(8 + x)$

b. $12x + 4$

c. $21x + 14$

d. $18 + 12x$

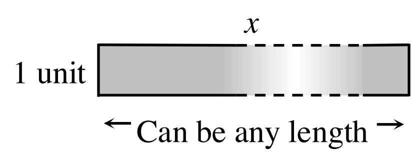
Simplifying With Zero

In the previous lessons, you simplified and rewrote algebraic expressions. In this lesson, you will continue to explore various ways to make expressions simpler by finding parts of them that make zero.

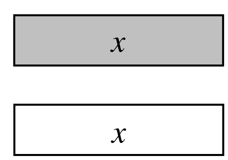
Zero is a relative newcomer to the number system. Its first appearance was as a placeholder around 400 B.C. in Babylon. The Ancient Greeks philosophized about whether zero was even a number: "How can nothing be something?" East Indian mathematicians are generally recognized as the first people to represent the quantity zero as a numeral and number in its own right about 600 A.D.

Zero now holds an important place in mathematics both as a numeral representing the absence of quantity and as a placeholder. Did you know there is no year 0 in the Gregorian calendar system (our current calendar system of 365 days in a year)? Until the creation of zero, number systems began at one.

17. When you use algebra tiles, +1 is represented with algebra tiles as a shaded small square and is always a positive unit. The opposite of 1, written –1, is an open small square and is always negative. Let's explore the variable x-tiles.



a. The variable x-tile is shaded, but is the number represented by a variable such as x always positive? Why or why not?

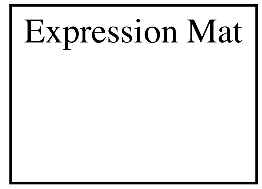


b. The opposite of the variable x, written $-x$, looks like it might be negative, but since the value of a variable can be any number (the opposite of –2 is 2), what can you say about the opposite of the variable x?

c. Is it possible to determine which is greater, x or $-x$? Explain.

d. What is true about $6 + (-6)$? What is true about $x + (-x)$ (the sum of a variable and its opposite)?

18. Get an Expression Mat Resource Page from your teacher, The mat will help you so you can tell the difference between the expression you are working on and everything else on your desk.



From your work in problem 34, you can say that situations like $6 + (-6)$ and $x + (-x)$ "create zeros." That is, when you add an equal number of tiles and their opposites, the result is zero. The pairs of unit tiles and x-tiles shown in that problem are examples of "zero pairs" of tiles.

Build each collection of tiles represented below on the mat. Name the collection using a simpler algebraic expression (one that has fewer terms). You can do this by finding and removing zero pairs and combining like terms. Note: A zero pair is two of the same kind of tile (for example, unit tiles), one of them positive and the other negative.

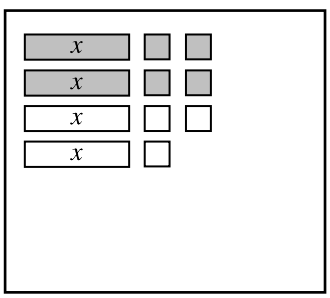
a. $2 + 2x + x + (-3) + (-3x)$

b. $-2 + 2x + 1 - x + (-5) + 2x$

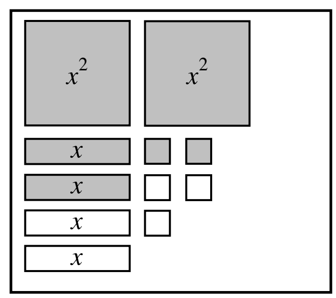
19. An **equivalent expression** refers to the same amount with a different name.

Build the Expression Mats shown in the pictures below. Write the expression shown on the expression mat, then write its simplified equivalent expression by making zeros (zero pairs) and combining like terms.

a.



b.



20. On your Expression Mat, build what is described below. Then write two different equivalent expressions to describe what is represented. One of the two representations should include parentheses.

a. The area of a rectangle with a width of 3 units and a length of $x + 5$.

b. Two equal groups of $3x - 2$.

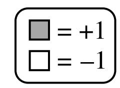
c. Four rows of $2x + 1$.

d. A number increased by one, then tripled.

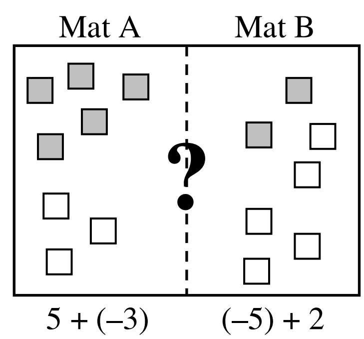
Comparing Expressions

Previously, you worked with writing and simplifying expressions. As you wrote expressions, you learned that it was helpful to simplify them by combining like terms and removing zeros. In this lesson, you and your teammates will use a tool for comparing expressions. The tool will allow you to determine whether one expression is greater than the other or if they are equivalent ways of writing the same thing (that is, if they are equal).

Remember that to represent expressions with algebra tiles, you will need to be very careful about how positives and negatives are distinguished. To help you understand the diagrams in the text, the legend at right will be placed on every page containing a mat. It shows the shading for +1 and –1. This model also represents a zero pair.



21. COMPARING EXPRESSIONS



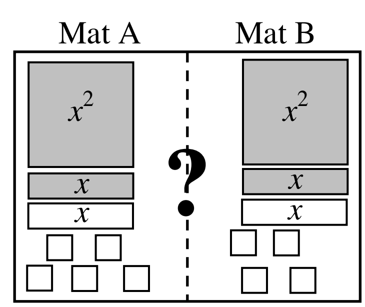
Ignacio and Oliver were playing a game. Each of them grabbed a handful of algebra tiles. They wanted to see whose expression had the greater value.

Two expressions can be compared by dividing the expression mat in half to change it into an **Expression Comparison Mat**. Then the two expressions can be built side by side and compared to see which one is greater.

- Oliver put his tiles on Mat A in the picture above and described it as 5 + (–3).
- Ignacio put his tiles on Mat B and said it was (–5) + 2.

With your team, find two different methods to simplify the two expressions so you can compare them. Which side of the mat is larger?

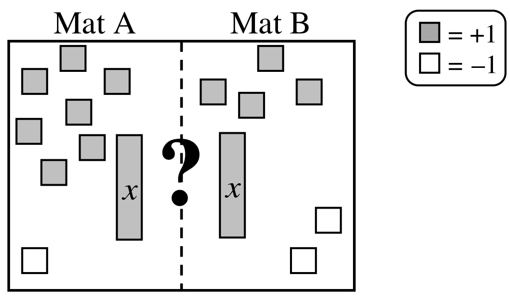
22. Using your Expression Comparison Mat, build the two expressions at right. Find a way to determine which side is greater, if possible. Show your work by sketching it on the Comparing Expressions Resource Page. Be ready to share your conclusion and your justification.



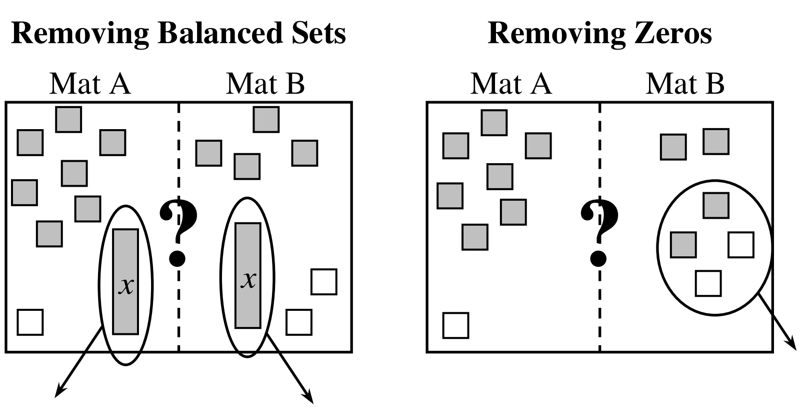
23. MORE COMPARING EXPRESSIONS—Is one expression greater?

Consider how you were able to compare the expressions in the previous problems. When is it possible to remove tiles to compare the expressions on the mats? In this problem, you will work with your team to identify two different "legal moves" for simplifying expressions.

Build the mat below using tiles and simplify the expressions. Record your work by drawing circles around the zeros or the balanced sets of tiles that you remove in each step on the Comparing Expressions Resource Page. Which expression is greater?



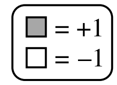
24. There are two kinds of moves you could use in problem 22 to simplify expressions with algebra tiles. First, you could remove matching (or balanced) sets of tiles from both sides of the mat. Second, you could remove zeros. Both moves are shown in the figures below. Justify why each of these moves can be used to simplify expressions.



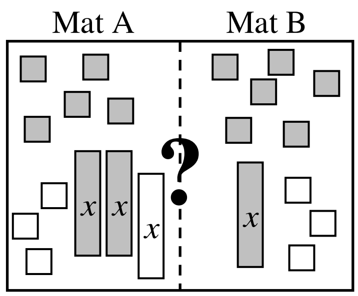
25. WHICH SIDE IS GREATER?

For each of the problems below, use the Comparing Expressions Resource Page and:

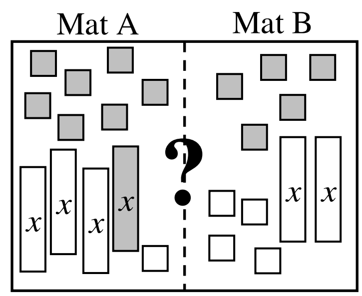
- Build the two expressions on your mat.
- Write an expression for each side below the mats for parts (a) through (d) OR draw the tiles in the space given on the resource page for parts (e) and (f).
- Use legal moves to determine which mat is greater, if possible. Record your work by drawing circles around the zeros or the balanced (matching) sets of tiles that you remove in each problem.



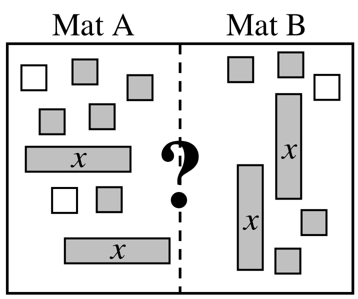
a.



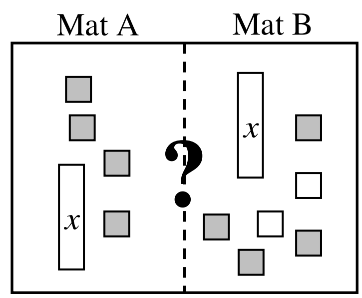
b.



c.



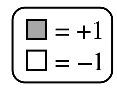
d.



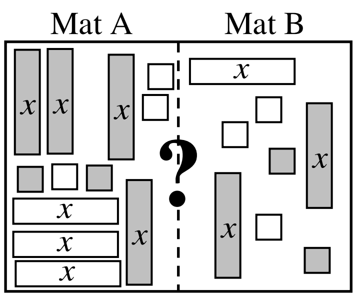
Comparing Quantities with Variables

Have you ever tried to make a decision when the information you have is uncertain? Perhaps you have tried to make plans on a summer day only to learn that it *might* rain. In that case, your decision might have been based on the weather, such as, “I will go swimming if it does not rain, or stay home and play video games if it does rain.” Sometimes in mathematics, solutions might depend on something you do not know, like the value of the variable. In this lesson you will study this kind of situation.

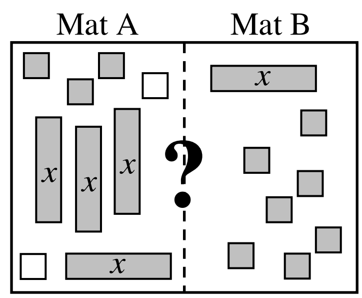
26. For each of the problems below, build the given expressions on your Expression Comparison Mat. Then use the simplification strategies of removing zeros and simplifying by removing matching pairs of tiles to determine which side is greater, if possible. Record your steps on the Simplify and Compare Resource Page.



a.

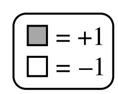


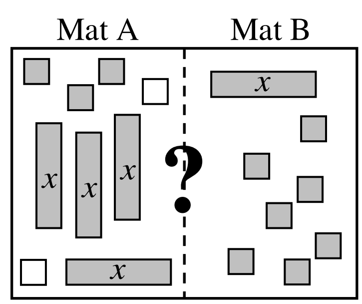
b.



27. WHAT HAPPENED?

When Ignacio and Oliver compared the expressions in part (b) of problem 25, they could not figure out which side was greater.





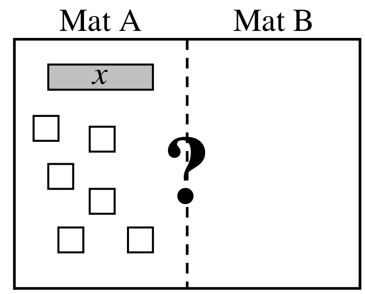
a. Is it always possible to determine which side of the Expression Comparison Mat is greater (has the greater value)? Why or why not? Be prepared to share your reasoning.

b. How is it possible for Mat A to have the greater value?

c. How is it possible for Mat B to have the greater value?

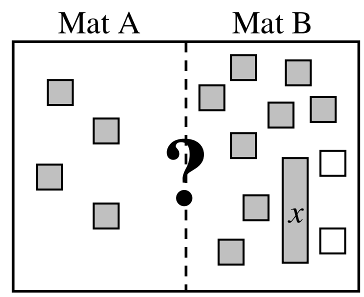
d. In what other way can Mat A and B be related? Explain.

28. Ignacio and Oliver are playing another game with the algebra tiles. After they simplify two new expressions, they are left with the expressions on their mats shown at right. They could not tell which part of the mat is greater just by looking.



a. One way to compare the mats is to separate the x-tiles and the unit tiles on different sides of the mat. Work with your team to find a way to have only x-tiles on Mat A. Make sure that you are able to justify that your moves are legal.

b. Using the same reasoning from part (a), what would you do to have only the variable on Mat B in the Expression Comparison Mat at right?



c. Write a short note to Ignacio and Oliver explaining this new strategy. Feel free to give it a name so it is easier for them to remember.

Solving Equations

In the last lesson, you figured out how to determine what values of x make one expression greater than another. In this lesson, you will study what can be learned about x when two expressions are equal.

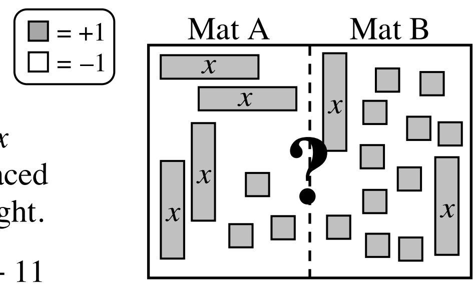
29. CHOOSING A PRICE PLAN

Sandeep works at a bowling alley that currently charges players \$3 to rent shoes and \$4 per game. However, his boss is thinking about charging \$11 to rent shoes and \$2 per game.

a. If a customer rents shoes and plays two games, will he or she pay more with the current price plan or the new price plan? Show how you know.

b. If the customer bowls 7 games, which price plan is cheaper?

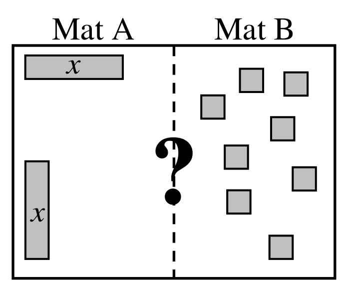
30. WILL THEY EVER BE EQUAL?



Sandeep decided to represent the two price plans from problem 47 with the expressions below, where x represents the number of games bowled. Then he placed them on the Expression Comparison Mat shown at right.

Original price: $4x + 3$ New price: $2x + 11$

a. Are his expressions correct? Find both the original and new prices when $x = 2$ and then again when $x = 7$ games.



b. Sandeep then simplified the expressions on the mat. What steps did Sandeep take to simplify the mat to this point?

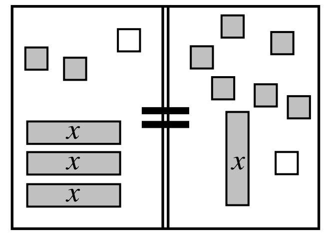
c. Sandeep noticed that for a certain number of games, customers would pay the same amount no matter which price plan his boss used. That is, he found a value of x that will make $4x + 3 = 2x + 11$. How many games would that customer bowl? What was the price he paid? Explain.

d. The value of x you found in part (c) is called a **solution** to the equation $4x + 3 = 2x + 11$ because it makes the equation true. That is, it makes both expressions have the same value.

Is $x = 6$ also a solution? How can you tell?

31. SOLVING FOR *X*

When the expressions on each side of the comparison mat are equal, they can be represented on a mat called an **Equation Mat**. Obtain an Equation Mat Resource Page and algebra tiles from your teacher. Now the "=" symbol on the central line indicates that the expressions on each side of the mat are equal.

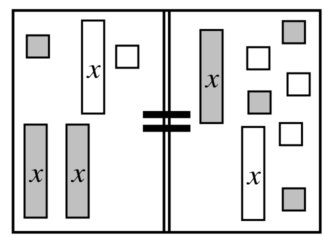


a. Build the equation represented by the Equation Mat at right on your own mat using algebra tiles.

b. On your paper, record the original equation represented on your Equation Mat.

c. Simplify the tiles on the mat as much as possible. Record what is on the mat after each legal move as you simplify each expression. What value of x will make the expressions equal?

32. Amelia wants to solve the equation shown on the Equation Mat at right. After she simplified each expression as much as possible, she was confused by the tiles that were left on the mat.



a. What was Amelia's original equation?

b. Remove any zero pairs that you find on each side of the Equation Mat. What happens?

c. What is the solution to this equation? That is, what value of x makes this equation true? Explain your reasoning.

Cases With Infinite or No Solutions

Are all equations solvable? Are all solutions a single number? Think about this: Annika was born first, and her brother William was born 4 years later. How old will William be when Annika is twice his age? How old will William be when Annika is exactly the same as his age?

In this lesson, you will continue to practice your strategies of combining like terms, removing zeros, and balancing to simplify and compare two expressions. You will also encounter unusual situations where the solution may be unexpected.

33. Many students believe that every equation has only one solution. However, in the introduction to this lesson you might have noticed that if Annika was four years older than her brother, William, they could never be the same age. Some situations have one solution, others have no solution, and still others have all numbers as solutions.

For each of the following equations, reason with your team to decide if the answer would be "One solution," "No solution," or "All numbers are solutions." If there is a single number solution, write it down. If you are not sure how many solutions there are, have each member of your team try a number to see if you can find a value that makes the equation work.

a. $x = x$

b. $x + 1 = x$

c. $x = 2x$

d. $x + x = 2 + x$

e. $x + x = x - x$

f. $x + x = 2x$

g. $x \cdot x = x^2$

h. $x - 1 = x$

34. SPECIAL CASES, Part One

Use the equation $8 + x + (-5) = (-4) + x + 7$ to complete parts (a) through (c).

a. Build the equation on your Equation Mat and simplify it as much as possible. Record your steps and what you see when you have simplified the equation fully. Draw a picture of your final mat.

b. Have each member of your team test a different value for x in the original equation, such as $x = 0$, $x = 1$, $x = -5$, $x = 10$, etc. What happens in each case?

c. Are there any solutions to this equation? If so, how many?

35. SPECIAL CASES, Part Two

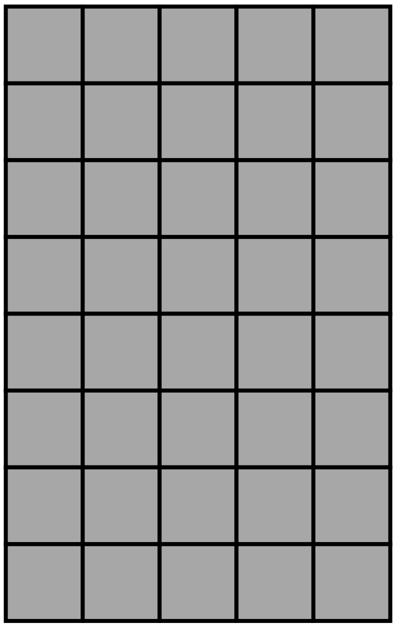
Use the equation $x + x + 2 = 2x$ to complete parts (a) through (c).

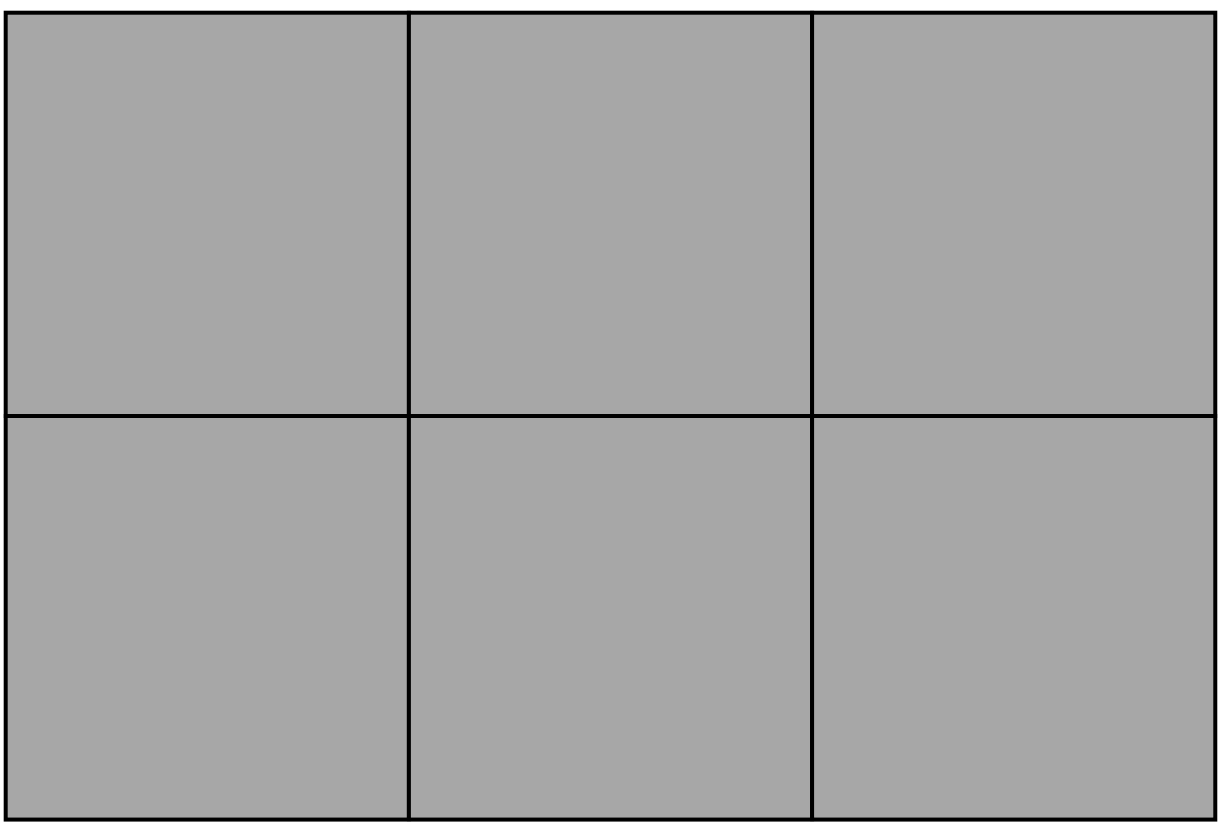
a. Build the equation on your Equation Mat and simplify it as much as possible. Record your steps and what you see when you have simplified the equation fully. Draw a picture of your final mat.

b. Have each member of your team test a different value for x in the equation, such as $x = 0$, $x = 1$, $x = -5$, $x = 10$, etc. What happens? Is there a pattern to the results you get from the equation?

c. Did you find any values for x that satisfied the equation in part (a)? When there is an imbalance of units left on the mat (such as $2 = 0$), what does this mean? Is $x = 0$ a solution to the equation?

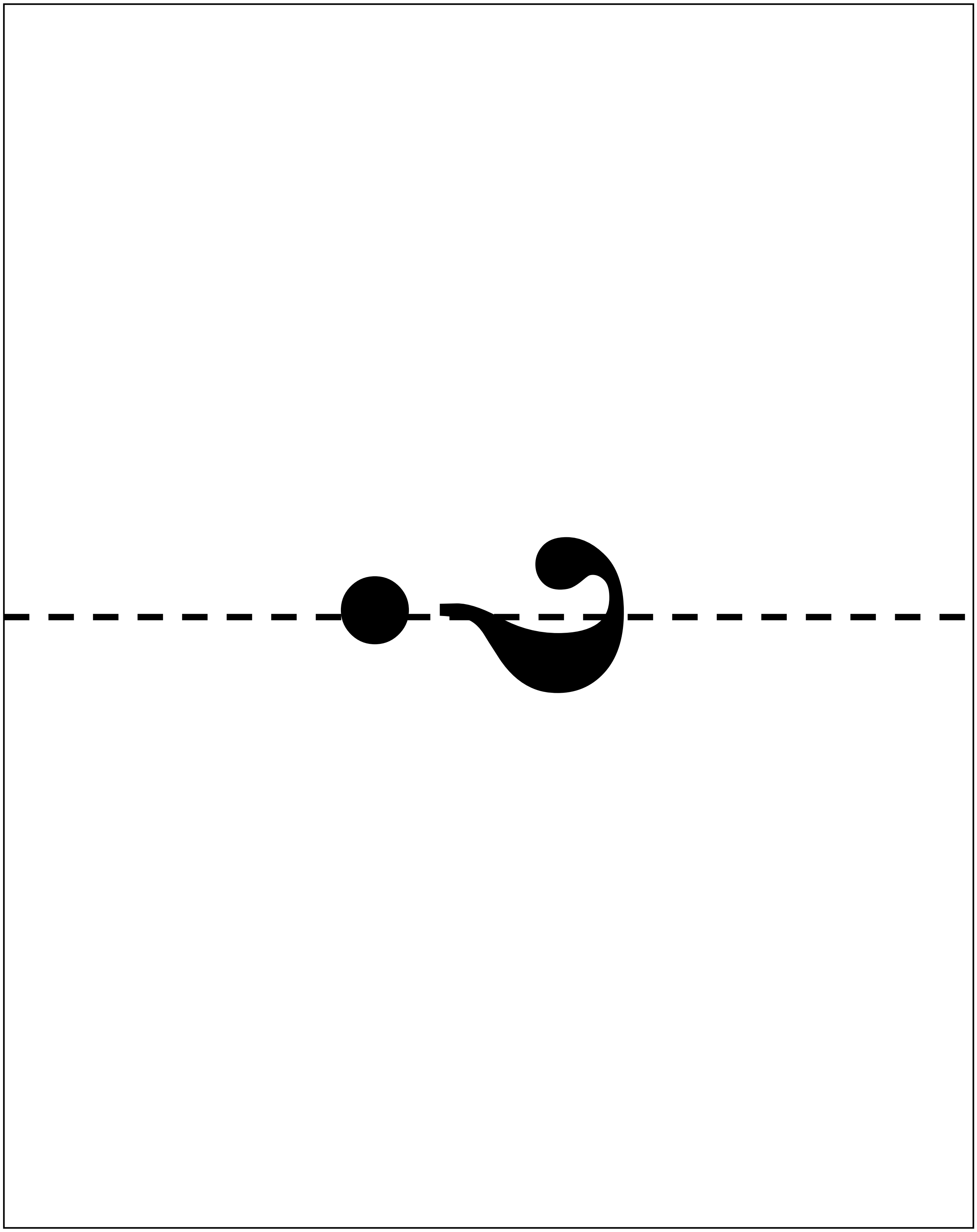
Algebra Tiles Resource Page



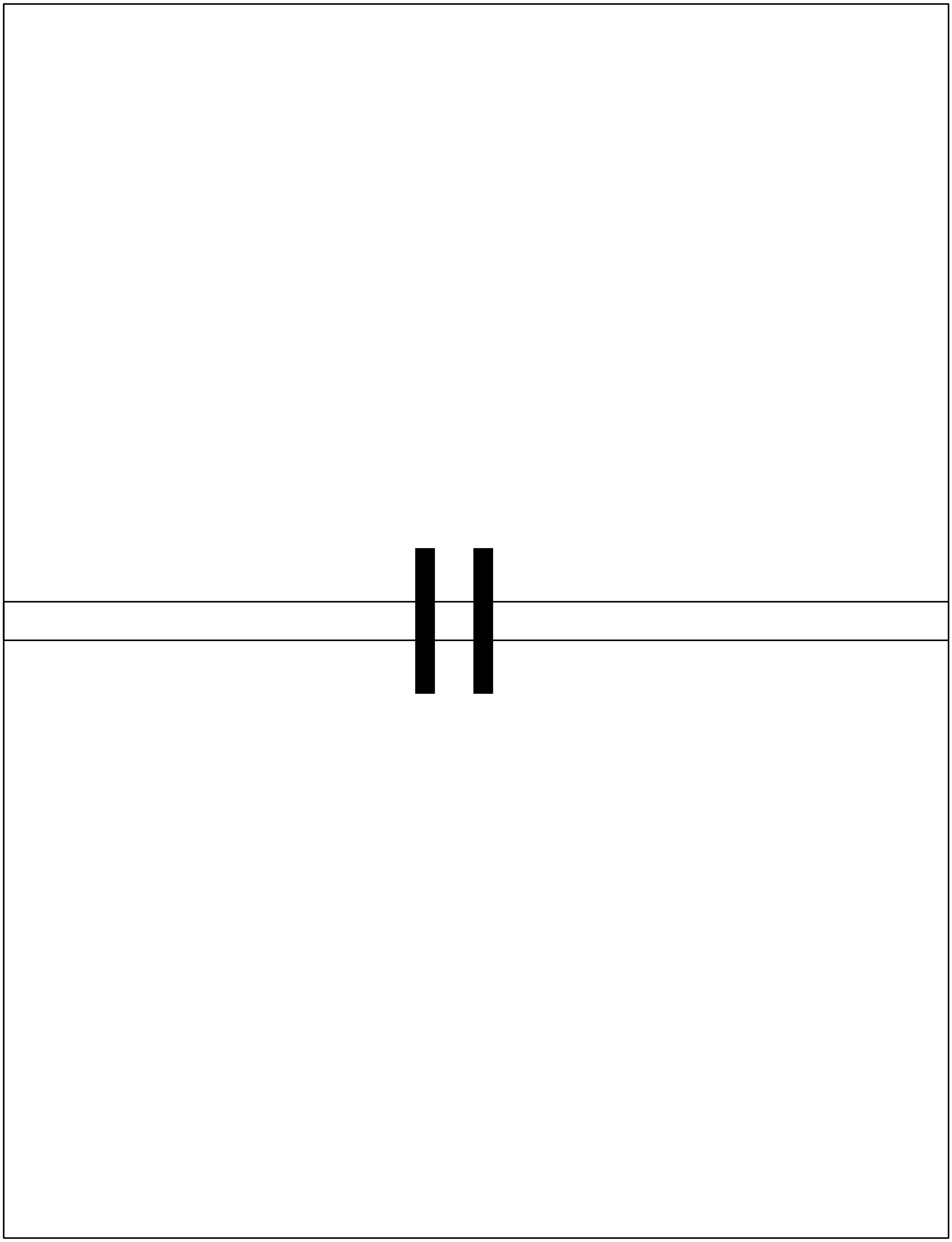


Expression Mat

Expression Comparison Mat



Equation Mat



Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 **Make sense of problems and persevere in solving them.**
- Find meaning in problems
- Look for entry points
- Analyze, conjecture and plan solution pathways
- Monitor and adjust
- Verify answers
- Ask themselves the question: "Does this make sense?"

2 **Reason abstractly and quantitatively.**
- Make sense of quantities and their relationships in problems
- Learn to contextualize and decontextualize
- Create coherent representations of problems

3 **Construct viable arguments and critique the reasoning of others.**
- Understand and use information to construct arguments
- Make and explore the truth of conjectures
- Recognize and use counterexamples
- Justify conclusions and respond to arguments of others

4 **Model with mathematics.**
- Apply mathematics to problems in everyday life
- Make assumptions and approximations to simplify a complicated situation
- Identify quantities in a practical situation
- Interpret results in the context of the situation and reflect on whether the results make sense

5 **Use appropriate tools strategically.**
- Consider the available tools when solving problems
- Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)
- Make sound decisions of which of these tools might be helpful

6 **Attend to precision.**
- Communicate precisely to others
- Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes
- Calculate accurately and efficiently

7 **Look for and make use of structure.**
- Discern patterns and structures
- Can step back for an overview and shift perspective
- See complicated things as single objects or as being composed of several objects

8 **Look for and express regularity in repeated reasoning.**
- Notice if calculations are repeated and look both for general methods and shortcuts
- In solving problems, maintain oversight of the process while attending to detail
- Evaluate the reasonableness of their immediate results