## Building on Foundations

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## Opening

## Building on Foundations

"When we are prepared to show our authentic awe and enthusiasm about mathematics and our genuine perplexity about challenges in mathematics, these actions become appropriate models for students to follow. Excitement is infectious! We can do this by learning with students rather than being the keeper of the answer key."

## Sign in and make a name tag

## Find participants who have the other pieces of the same graph. As a team, come up with a story that could be represented by your team's graph.

## Building on Foundations



More Math For More People

## Opening

Housekeeping

+ Bathrooms
+ 8:00 AM - 4:00 PM
+ Breaks scheduled and as needed
+ Lunch
+ Parking Lot Poster
+ Supply/Resource Table



## Opening

Learning Event eBook Access

Account ManagementeBook Licensing System
( $)$ Shop
(V) Use Enrollment Pin

## eBooks Access

 my.cpm.org XXXXXX
## Opening

## Outcomes

## Participants will...

Consider how the mathematical storyline provides opportunities to foster curiosity, make connections, and deepen understanding.

Purposefully use the embedded resources to plan for sharing math authority with students.

Reflect on past year(s) of teaching CPM curriculum and consider ways to enhance your collaborative classroom.

Develop an action plan to support your ongoing professional learning.

Agenda
Building on Foundations

$\square$ Opening \& Icebreaker
$\square$ Mathematical Storyline
$\square$ Sharing Math Authority
$\square$ Creating Collaborative Classrooms
$\square$ Closure

## Opening

> Research Connections
> CPM Implementation Progress Tool

# The three pillars represent researched best practice in math education around which the CPM program is designed. 

## Collaborative Learning

Research says students learn ideas more deeply when they discuss ideas with classmates.

Problem-Based Learning
Research says students learn ideas more usefully for other arenas when they learn by attacking problems.

Mixed, Spaced Practice
Research says students learn ideas more permanently when they are required to engage and re-engage with those ideas for months or even years.

Welcome

Working Agreements

Be willing to take risks. Have a visionary mindset. Stay engaged.
Explore and reflect on your beliefs. Give grace to others and yourself.

Change takes time, effort, and support!

Agenda
Building on Foundations


Icebreaker
The Joy of Teaching

# PERSONALIZE WITH PICTURES TO REPRESENT YOUR TEACHING STORY. 

Icebreaker
The Joy of Teaching

## Team Task:

1. Introduce yourself to your team.
2. Share the story of how you became a teacher.

## Agenda

Building on Foundations

Mixed, Spaced Practice
Research says students learn ideas more permanently when they are required to engage and re-engage with those ideas for months or even years.

## $\square$ Opening \& Icebreaker $\square$ Mathematical Storyline $\square$ Sharing Math Authority $\square$ Creating Collaborative Classrooms $\square$ Closure

Outcome: Consider how the mathematical storyline provides opportunities to foster curiosity, make connections, and deepen understanding.

## Mathematical Storyline

Icebreaker: CPM Lesson

## Team Task: (10 minutes)

Reflect over the last year, which CPM lesson did you and your students find engaging and joyful? Why?

Team Roles - Assigned alphabetically by first name.
Resource Manager - make sure everyone has an opportunity to share
Facilitator - start your team by being the first to share
Recorder/Reporter - look for commonalities and be prepared to share out
Task Manager - keep track of time and update your team as necessary

## Mathematical Storyline

Does the sequence of events matter?

## Think-Pair-Share

Think of a story using the illustrations.
Pair with your elbow partner to co-create a story.

Share stories as team and consider these questions:.

```
+ What if you took out cell C?
+ What if you removed the last cell in your story?
+ What would you draw to "end" the story?
```

Resource Manager - get two envelopes and be sure the questions are discussed

Facilitator - lead and support the team's discussion

Recorder/Reporter - be sure both stories are shared

Task Manager - keep track of time

## Mathematical Storyline

## Does the sequence of events matter? - Debrief


(The Mathematics Teacher, Dietiker, November 2016) Used with permission from the publisher for CPM Learning Events. May not be reproduced or redistributed by others without permission.

## Mathematical Storyline <br> Mathematical Story Defined

"Seeing how mathematics unfolds, connecting a beginning to an ending, so that somebody who is experiencing it can kind of sense a storyline and predict where it's going."


## Mathematical Storyline

## Experience a Mathematical Story: CC1 Lesson 7.2.2

Method 1: Using diagrams
To divide any number by a fraction using a diagram, create a model of the situation using rectangles, a linear model, or some visual representation of it. Then break that model into the fractional parts named.

For example, to divide $\frac{7}{8} \div \frac{1}{2}$, you can draw the diagram at right to visualize how many $\frac{1}{2}$
-sized pieces fit into $\frac{7}{8}$. The diagram shows that one $\frac{1}{2}$ fits one time, with $\frac{3}{8}$ of a whole left. Since $\frac{3}{8}$ is $\frac{3}{4}$ of $\frac{1}{2}$, you can see that $1 \frac{3}{4} \frac{1}{2}$-sized pieces fit into $\frac{7}{8}$, so $\frac{7}{8} \div \frac{1}{2}=1 \frac{3}{4}$.

Alternately, you could think of $\frac{7}{8}$ as the quantity that you have and $\frac{1}{2}$ as the size of the group that you want, such as having $\frac{7}{8}$ ounces of chocolate and needing $\frac{1}{2}$ ounce for each cake recipe. How many cakes could you make? In this case, the diagram at right might be useful. The diagram shows $\frac{7}{8}$ being divided into groups of $\frac{1}{2}$. The leftover $\frac{3}{8}$ ounces creates another $\frac{3}{4}$ of a group, so again, $\frac{7}{8} \div \frac{1}{2}=1 \frac{3}{4}$.


Math Notes

Method 2: Using Common Denominators
To divide a number by a fraction using common denominators, express both numbers as fractions with $\quad \frac{2}{5} \div \frac{3}{10}=\frac{4}{10} \div \frac{3}{10}$ the same denominator. Then divide the first numerator by the second. An example is shown at right.

$$
=4 \div 3
$$

$$
=\frac{4}{3}=1 \frac{1}{3}
$$

7-53. Calculate each of the following products. Homework Help
a. $\frac{1}{8} \cdot \frac{8}{1}$
b. $\frac{3}{4} \cdot \frac{4}{3}$
c. $\frac{2}{3} \cdot \frac{3}{2}$
d. $7 \cdot \frac{1}{7}$
e. What do the products in parts (a) through (d) have in common?

## Mathematical Storyline

## Experience a Mathematical Story

### 7.2.3 How can I divide?

## Division with Fractions and Decimals



This lesson will bring you more division strategies! You will continue your work with dividing fractions to include a new strategy for dividing by fractions. You will also extend your knowledge to division of decimals.

## Mathematical Storyline

## Experience a Mathematical Story

## Your Task:

+ Examine the problems and the mathematical storyline.
+ Change the order of the problems.
- How does the sequence foster curiosity for students?
- How does changing the sequence affect the story?

Find the missing number in each puzzle.
i. 6 $\qquad$ $=1$
ii. $4 \cdot \ldots=1$
iii. $2 / 3 \cdot \ldots=1$

Malik was working on the division problem $5 \div 3 / 4$.

Copy the expression and simplify it.

Cheryl used the problem $1 / 6 \div 3 / 4$.

Copy and complete
Cheryl's calculation.

## Mathematical Storyline

## Experience a Mathematical Story: CC1 Lesson 7.2.3

## Team Task: (15 minutes)

Complete 7-57 through 7-59.
Discuss the following.

+ How does the authors' chosen sequence provide opportunities for surprise and intrigue?
+ What are the mathematical characters, settings, and actions?
+ Where/What was the a-ha of this lesson? How does this connect to the lesson goal? How can you assess student understanding of the goal?

Facilitator - lead the team discussion
Resource Manager - ensure two eBooks are open and everyone has access to the problems

Recorder/Reporter - make sure all voices are heard

Task Manager - make sure everyone stays together while working through the problems

Mathematical Storyline
Debrief the Storyline

## Walk and Talk

How does the authors' chosen sequence provide opportunities for surprise and intrigue?
What are the mathematical characters, settings, and actions?
Where/What was the "a-ha" of this lesson? How does this connect to the lesson goal? How can you assess student understanding of the goal?

## Break



## Mathematical Storyline

Definition

"Enacting a sequence so that it offers dramatic conflict can also offer students a memorable experience and, since it increases student attention, lead to more learning by students (Egan, 1989)."

Mathematics Teacher, NCTM, November 2016

## Mathematical Storyline

## Scenarios

## Four Corners Jigsaw

Independently read the scenario and two questions. (2 min)
Discuss with your corner. Be prepared to share with your team. (6 min)


## Mathematical Storyline

Scenarios

## Team Task: (15 minutes)

Review team roles.
Consider the following questions.

+ What is the impact of modifying the storyline?
+ How does teacher facilitation affect the storyline?

Resource Manager - be prepared to share out your team's thinking
Facilitator - start the discussion by sharing Teacher A's scenario
Recorder/Reporter - ensure all team members record their thoughts
Task Manager - keep track of time and update your team as necessary

## Mathematical Storyline

Effects on the Mathematical Storyline

## Mathematical Storyline Scenarios



How was this lesson affected by the modification?

## Mathematical Storyline <br> Reflection

## Dyad

It is important to develop this skill at the conceptual level because...
How will this concept come back in your course?

$$
\frac{\frac{11}{8}}{\frac{3}{4}} \cdot \frac{4}{\frac{3}{3}}=\frac{\frac{44}{24}}{1}=\frac{44}{24}=1 \frac{20}{24}=1 \frac{5}{6}
$$

## Mathematical Storyline <br> Reflection

# CPM Tutorials CPM Help CPM Links * 

+ Search for "Giant One" in your eBook.
+ Click on the "Giant One" search results that occur in the Teacher Notes.
$+\quad$ Note: The CCG and Stats eBook do not contain the "Giant One" explicitly in the search results.
+ Share how the "Giant One" is incorporated in different courses.


## Teacher

Notes
8.2.2
8.2.3 use the Giant One to simplify expressions with negativ as the denominator of a fraction and then simplifying in problem $8-\delta$

Some teams may create an equivalent ratio by multiplying by a Gian factor relationship between the different lengths. Students may also s denominator.

## Mathematical Storyline

## Research Connections <br> CPM Implementation Progress Tool

SECTION ONE: The pillars that represent necessary first steps in any implementation.

## Collaborative Learning

Students and teachers are aware of the purpose for and value of working in teams, and are familiar with team norms and roles.

## Problem-Based Learning

Students and teachers share math authority as they value and engage in productive struggle. Teachers guide without taking over the thinking.

## Mixed, Spaced Practice

Both individual lessons and chapters are followed, using suggested pacing. Review \& Preview problems are assigned and valued as an essential part of learning.

## Mathematical Storyline

Mathematical Story Defined

## MORE MATH $\xrightarrow{ }$ <br> FOR MORE PEOPLE

Episode 13: Where Joel and Misty make a new friend with Dr. Leslie Dietiker and talk about Mathematical Storylines

## Mathematical Storyline

## Mathematical Story Defined

"Understanding how to read for mathematical stories across tasks enables teachers to read their own math curriculum materials for hidden opportunities of surprise or wonder and then capitalize on these opportunities in class. Taking advantage of these opportunities can trigger interest in students, which when repeated can lead to the development of their personal interest in mathematics (Hidi and Renninger 2006)."

## Mathematical Storyline

## Planning Time

## Collaborate with a course-alike partner, trio, or team.

## Your Task:

+ Reflect on the course's recurring characters, themes, and settings.
+ Select a challenging lesson.
- Reflect on the mathematical storyline of the lesson.
- Discuss the lesson's launch, explore, and closure.

Consider how the structure of the lesson...

+ maintains the cognitive demands of the task
+ supports students' ability to take charge of their learning and become more independent
$+\quad$ allows students to summarize their learning and assess their understanding
$+\quad$ creates opportunities for surprise or wonder


# Sharing Math Authority 

Building on Foundations

Problem-Based Learning
Research says students learn ideas more usefully for other arenas when they learn by attacking problems.

## $\square$ Opening \& Icebreaker $\square$ Mathematical Storyline VSharing Math Authority $\square$ Creating Collaborative Classrooms $\square$ Closure

Outcome: Purposefully use the embedded resources to plan for sharing math authority with students.

## Sharing Math Authority

The Elements

## "The implications are clear. Curriculum matters, but how teachers use curriculum matters even more."

The Elements: Transforming Teaching through Curriculum-Based Professional Learning -Carnegie Corporation of New York, 2020

## Sharing Math Authority

Research Connections - Why?

## Proximity Dyad

> How do the practices provide the opportunity for you to co-create the mathematical story with your students?

## Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

## Sharing Math Authority

CPM Course Design
L
CPM Course Design


Implement tasks that promote reasoning and

Build procedural fluency from conceptual problem solving understanding


## Sharing Math Authority

Looks Like, Sounds Like, Feels Like Y-chart

## 02:00

Facilitator - make sure everyone is heard
Resource Manager - get materials for activity
Recorder/Reporter - start with the marker and be sure it rotates after 2 minutes

Task Manager - keep track of time

Sharing Math Authority


## Sharing Math Authority

## An Upside Down Approach

## CPM: STUDENT CENTERED PROBLEM-BASED LEARNING



## Sharing Math Authority

## Desmos Reflection

## Go to student.desmos.com and type in:

\#\#\# \#\#\#

Source: NCTM,
Navigating the Uncertainty of
Sharing Mathematical Authority

## Sharing Math Authority <br> Share Your Experience <br> 06:00

## Elevator Talk

Team Task: Each team member will select one prompt to answer.

+ Share a specific example of sharing the math authority with students.
+ Share how your students respond when you shared the math authority.
+ Share how it feels to share the math authority.
+ Share what is challenging about sharing the math authority.
+ Share what expectations might be helpful when sharing math authority.


# Sharing Math Authority 

## Research Connections <br> CPM Implementation Progress Tool

SECTION ONE: The pillars that represent necessary first steps in any implementation.

## Collaborative Learning

Students and teachers are aware of the purpose for and value of working in teams, and are familiar with team norms and roles.

## Problem-Based Learning

Students and teachers share math authority as they value and engage in productive struggle. Teachers guide without taking over the thinking.

Mixed, Spaced Practice
Both individual lessons and chapters are followed, using suggested pacing. Review \& Preview problems are assigned and valued as an essential part of learning.

## Lunch Time

## Return by:


@CPMmath
\#MoreMathforMorePeople

## Opening

## CPM’s Equity Principles

The goal of teaching is to help all students transition from dependent to independent learners.

Relationships are of vital importance.

Student uniqueness is an asset, not a deficit.

Reflection is a crucial part of growth.

# Sharing Math Authority 

## Research Connection <br> CPM Implementation Progress Tool

SECTION TWO: Features of desired student learning when the pillars are in place.

| Collaborative Learning | Problem-Based Learning | Mixed, Spaced Practice |
| :--- | :--- | :--- |
| Students read and make sense of <br> problems together. | Student thinking at varied depths <br> of conceptual understanding are <br> openly shared and valued. | Students work through lessons at <br> an appropriate pace. |
| Students are able to listen to the <br> ideas of others and communicate <br> their own ideas both in teams and <br> during whole class discussions. | Students demonstrate and value <br> both conceptual and procedural <br> knowledge. | Students understand that mastery <br> takes time, effort, and support. |
| Students listen carefully to the | Students look for, compare, and <br> connect multiple models and | Students are aware of learning <br> targets and periodically self-assess <br> thing of others and respond with <br> clarifying questions or extensions of progress towards those <br> solution strategies. |
| their own. |  |  |

> Which student actions are evident when you share the math authority?

What actions are happening in your classroom?

Sharing Math Authority
Self Reflection

## Two Stars and a Wish

What is going well?
What is a challenge?


## Sharing Math Authority

## Resources Concept Map

Explore your assigned resources. ( 10 min )
As a team, create a concept map with the theme of Sharing Math Authority. ( 20 min )


Facilitator: Universal Access, Pocket Questions, Suggested Lesson Plan, Lesson Plan Structure: Launch


Resource Manager: Chapter Opening, Skill Builders (Supplemental Resources), Math Notes, Lesson Plan Structure: Explore

- 

Recorder/Reporter: Lesson Mathcast, Teambuilders/ Icebreakers, Lesson Plan Structure: Closure, Homework Help


Task Manager: Team Roles, Classroom/Team Expectations, Chapter Closure, Checkpoints


## How do these resources support sharing math authority?

 What connections do you see to the Mathematical Teaching Practices?
## Sharing Math Authority

## Resources Concept Map



## Hosted Gallery Walk

## Consider how these resources might support you and your students.

Record ideas/resources to add to your action plan.

## Recorder/Reporter:

Share your team's connections.
What embedded CPM resources support sharing math authority?


Sharing Math Authority Debrief

## Embedded CPM Resources

Suggested Lesson Plan

Study Team \& Teaching Strategies

Pocket Questions

Mathcast
Universal Access
Lesson Plan
Structure:
(Launch, Explore, Closure)

Learning Logs

| Suggested Lesson Plan | Universal Access |  |
| :---: | :---: | :---: |
| Study Team \& Teaching Strategies | Lesson Plan Structure: (Launch, Explore, Closure) | Classroom/Team Expectations <br> Chapter Opening <br> Chapter Closure |

## Break



## Collaborative Classrooms

Building on Foundations

Collaborative Learning
Research says students learn ideas more deeply when they discuss
ideas with classmates.

## $\square$ Opening \& Icebreaker $\square$ Mathematical Storyline VSharing Math Authority $\square$ Creating Collaborative Classrooms $\square$ Closure

Outcome: Reflect on past year(s) of teaching CPM curriculum and consider ways to enhance your collaborative classroom.

## Collaborative Classrooms

Teambuilder: Let’s Make Squares

Team task: 10-15 minutes
Use all 12 strips each time you make $1,2,3, \ldots 12$ squares.
No Cuts or Bends: Strips must lay flat on the table.
Three Squares


No Stacking: Strips can cross any other strips, but they cannot lay on top of, or cover up, one another lengthwise.

No Touching: Strips cannot lay side by side with edges touching.

## Teambuilders/Icebreakers

(Teacher tab $\rightarrow$ Team Support $\rightarrow$ Team Resources $\rightarrow$ under Team Strategies)

Four Squares


## Collaborative Classrooms

Let’s Make Squares - Debrief

## Reflect on your team's collaboration.



What does collaboration not look like?


What does collaboration look like?

Create a Collaboration Rubric
Idea from Building Thinking Classrooms

## Collaborative Classrooms

## Visibly Random Teams

## Let's practice collaboration with new teams!



PETER LILJEDAHL
FOREWORD BY TRAC JOHMSTOW ZHELR flustrations br Laurh wheeler

https://pickerwheel.com/tools/random-team-generator/

## Collaborative Classrooms

## Team Roles

## Assign team roles: Alphabetically by first name

Facilitator - Alphabetically first<br>Resource Manager - Alphabetically second<br>Recorder/Reporter - Alphabetically third<br>Task Manager - Alphabetically fourth

## Collaborative Classrooms

Icebreaker

Team Task: On the VNPS, each person will draw something joyful.


## Collaborative Classrooms

Vertical, Non-Permanent Surface


## Suggested Expectations:

+ Only write the ideas of others.
+ Only erase your own ideas.
+ When the timer sounds, the marker passes to a different person.

Just because sitting and writing in the notebook is the obvious place for some activities, it does not have to be the workspace for all activities. -Peter Liljedahl

## Collaborative Classrooms

Mathematical Storyline



## Chapter 8

## Quadratic Functions



## Collaborative Classrooms

Mathematical Storyline: Section 8.1, Factored Form
In this section, students develop a method to change a quadratic equation written as a sum into its product form (also called its factored form).


Area as a Sum:

$$
6 x^{2}+17 x+5
$$

Area as a Product:

$$
(3 x+1)(2 x+5)
$$


$(2 x+5)$

## Collaborative Classrooms

Mathematical Storyline: Section 8.1, Factoring Process
Students use the generic rectangle to factor quadratic equations.


Product $=a b c d$

$$
\text { Product }=a b c d
$$

## Collaborative Classrooms

## Mathematical Storyline: Section 8.2

Students generate each representation of a quadratic function (rule, graph, table, and situation). They also develop a method to find the $x$-intercepts of a parabola using the Zero Product Property and use square roots to find the $x$-intercepts.

Finally, they "complete the square."


## Collaborative Classrooms

## CCA Lesson 8.2.5

Completing the Square

## Math Goal:

Learn how to convert the equation of a parabola into graphing form.

Team Goal:
Utilize your VNPS to make your team's thinking visible. Use the collaboration rubric to guide your teamwork.


## Collaborative Classrooms

Debrief CCA Lesson 8.2.5

## Independent Reflection

Review the CCA Lesson 8.2.5 Teacher Notes (eBook).
Compare the Teacher Notes with the Launch-Explore-Closure Lesson Plan.

Reflect using the Implementation Progress Tool.

+ What did I do, as the teacher, while you were working?
+ What was your experience on the role of the student?
Additional Challenge:
How do you see these math concepts appear in the CPM courses' storyline?


## Collaborative Classrooms

Debrief CCA Lesson 8.2.5
Think-Ink-Pair-Share

## Think-Ink-Pair-Share

+ How was your experience as a student during that lesson?
+ How did the lesson compare to the Teacher Notes?
+ What teacher moves supported your collaboration and engagement?


## Collaborative Classrooms

## Research Connections <br> CPM Implementation Progress Tool

SECTION ONE: The pillars that represent necessary first steps in any implementation.

| Collaborative Learning |
| :--- |
| Students and teachers are aware <br> of the purpose for and value of <br> working in teams, and are familiar <br> with team norms and roles. |

Problem-Based Learning<br>Students and teachers share math authority as they value and engage in productive struggle. Teachers guide without taking over the thinking.

Mixed, Spaced Practice
Both individual lessons and chapters are followed, using suggested pacing. Review \& Preview problems are assigned and valued as an essential part of learning.

## Collaborative Classrooms

Planning Time

## Collaborate with a course-alike partner, trio, or team.

## Your Task:

+ Select a lesson from your course.
+ Reflect on how you will share math authority and encourage collaboration during the lesson.
+ Discuss the launch, explore, and closure of your lesson.

Consider how your lesson plan will:

+ incorporate embedded CPM resources
+ assess the effectiveness of collaboration of your teams
+ support students' ability to take charge of their learning and become more independent


## Closure

Building on Foundations

$\square$ Opening \& Icebreaker $\square$ Mathematical Storyline $\square$ Sharing Math Authority $\square$ Creating Collaborative Classrooms $\square$ Vlosure

Outcome: Develop an action plan to support your ongoing professional learning.

## Closure

Outcomes

## Participants...

Consider how the mathematical storyline provides opportunities to foster curiosity, make connections, and deepen understanding.

Purposefully use the embedded resources to plan for sharing math authority with students.

Reflect on past year(s) of teaching CPM curriculum and consider ways to enhance your collaborative classroom.

Develop an action plan to support your ongoing professional learning.

## Closure

Professional Learning Portal

## Continue your story...

## Enroll in the On-Demand Module

$\rightarrow$ On-Demand Modules
$\rightarrow$ Additional Teacher Resources
$\rightarrow$ Building on Foundations On-Demand
Module
$\rightarrow$ Connections to Foundations for Implementation

## Action Plans

Learning Log
(․). File Cabinet
(12) Course Content Sessions
((D) Live Virtual Sessions

弾 On-Demand Learning
(Content, Instructional, Inclusion)

## Closure

+ Parking Lot
+ Attendance \& Feedback
Either scan the QR code OR
Enter passcode in the Portal XXXXXXX
+ Continuing Education Credit


