Introduction

As we move into the second decade of the twenty-first century, one thing is clear: Our country needs highly trained workers who can wrestle with complex problems. Gone are the days when basic skills could be counted on to yield high-paying jobs and an acceptable standard of living. Especially needed are individuals who can think, reason, and engage effectively in quantitative problem solving.

The instructional practices used in the majority of our nation's classrooms will not prepare students for these new demands. National studies have shown that American students are not routinely asked to engage in conceptual thinking or complex problem solving (Stigler and Hiebert 1999). Most schoolwork consists of assignments composed of "problems" for which students have been taught a preferred method of solving. There is little engagement of student "thinking" in such tasks, only the straightforward application of previously learned skills and recall of memorized facts. It is unrealistic to expect students to learn to grapple with the unstructured, messy challenges of today's world if they are forced to sit silently in rows, complete basic skills worksheets, and engage in teacher-led "discussions" that consist of literal, fact-based questions and answers.

What kind of learning experiences *will* prepare students for the demands of the twenty-first century? *Research tells us that complex knowledge and skills are learned through social interaction* (Vygotsky 1978; Lave and Wenger 1991). We learn through a process of knowledge construction that requires us to actively manipulate and refine information and then integrate it with our prior understandings. Social interaction provides us with the opportunity to use others as resources, to share our ideas with others, and to participate in the joint construction of knowledge. In mathematics classrooms, high-quality discussions support student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas. These are all important features of what it means to be "mathematically literate."

Creating discussion-based opportunities for student learning will require learning on the part of many teachers. First, teachers will need to learn how to select and set up cognitively challenging instructional tasks in their classrooms, since such high-level tasks provide the grist for worthwhile discussions. Over the years, however, most textbooks have fed teachers a steady diet of routine, procedural tasks around which it would be difficult, if not impossible, to organize an engaging discussion.

Second, teachers must learn how to support their students as they engage with and discuss their solutions to cognitively challenging tasks. We know from our own past research that once high-level tasks are introduced in the classroom, many teachers have difficulty maintaining the cognitive demand of those tasks as students engage with them (Stein, Grover, and Henningsen 1996). Students often end up thinking and reasoning at a lower level than the task is intended to elicit. One of the reasons for this is teachers' difficulties in orchestrating discussions that productively use students' ideas and strategies that are generated in response to high-level tasks.

A typical lesson that uses a high-level instructional task proceeds in three phases. It begins with the teacher's launching of a mathematical problem that embodies important mathematical ideas and can be solved in multiple ways. During this "launch phase," the teacher introduces students to the problem, the tools that are available for working on it, and the nature of the products that the students will be expected to produce. This phase is followed by the "explore phase," in which students work on the problem, often discussing it in pairs or small groups. As students work on the problem, they are encouraged to solve it in whatever way makes sense to them and be prepared to explain their approach to others in the class. The lesson then concludes with a whole-class discussion and summary of various student-generated approaches to solving the problem. During this "discuss and summarize" phase, a variety of approaches to the problem are displayed for the whole class to view and discuss.

Why are these end-of-class discussions so difficult to orchestrate? Research tells us that students learn when they are encouraged to become the authors of their own ideas and when they are held accountable for reasoning about and understanding key ideas (Engle and Conant 2002). In practice, doing both of these simultaneously is very difficult. By their nature, high-level tasks do not lead all students to solve the problem in the same way. Rather, teachers can and should expect to see varied (both correct and incorrect) approaches to solving the task during the discussion phase of the lesson. In theory, this is a good thing because students are "authoring" (or constructing) their own ways of solving the problem.

The challenge rests in the fact that teachers must also align the many disparate approaches that students generate in response to high-level tasks with the learning goal of the lesson. It is the teachers' responsibility to move students collectively toward, and hold them accountable for, the development of a set of ideas and processes that are central to the discipline—those that are widely accepted as worthwhile and important in mathematics as well as necessary for students' future learning of mathematics in school. If the teacher fails to do this, the balance tips too far toward student authority, and classroom discussions become unmoored from accepted disciplinary understandings.

The key is to maintain the right balance. Too much focus on accountability can undermine students' authority and sense making and, unwittingly, encourage increased reliance on teacher direction. Students quickly get the message—often from subtle cues—that "knowing mathematics" means using only those strategies that have been validated by the teacher or textbook; correspondingly, they learn not to use or trust their own reasoning. Too much focus on student authorship, on the other hand, leads to classroom discussions that are free-for-alls.

Successful or Superficial? Discussion in David Crane's Classroom

In short, the teacher's role in discussions is critical. Without expert guidance, discussions in mathematics classrooms can easily devolve into the teacher taking over the lesson and providing a "lecture," on the one hand, or, on the other, the students presenting an unconnected series of show-and-tell demonstrations, all of which are treated equally and together illuminate little about the mathematical ideas that are the goal of the lesson. Consider, for example, the following vignette (from Stein and colleagues [2008]), featuring a fourth-grade teacher, David Crane.

ACTIVE ENGAGEMENT 0.1

As you read the Case of David Crane, identify instances of student authorship of ideas and approaches, as well as instances of holding students accountable to the discipline.

Leaves and Caterpillars: The Case of David Crane

Students in Mr. Crane's fourth-grade class were solving the following problem: "A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?" Mr. Crane told his students that they could solve the problem any way they wanted, but he emphasized that they needed to be able to explain how they got their answer and why it worked.

As students worked in pairs to solve the problem, Mr. Crane walked around the room, making sure that students were on task and making progress on the problem. He was pleased to see that students were using many different approaches to the problem—making tables, drawing pictures, and, in some cases, writing explanations.

He noticed that two pairs of students had gotten wrong answers (see fig. 0.1). Mr. Crane wasn't too concerned about the incorrect responses, however, since he felt that once several correct solution strategies were presented, these students would see what they did wrong and have new strategies for solving similar problems in the future.

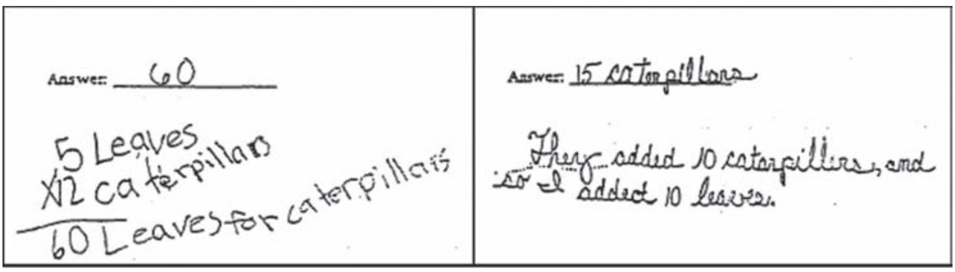

Fig. 0.1. Solutions produced by Darnell and Marcus (left) and Missy and Kate (right)

When most students were finished, Mr. Crane called the class together to discuss the problem. He began the discussion by asking for volunteers to share their solutions and strategies, being careful to avoid calling on the students with incorrect solutions. Over the course of the next 15 minutes, first Kyra, then Jason, Jamal, Melissa, Martin, and Janine volunteered to present the solutions to the task that they and their partners had created (see fig. 0.2). During each presentation, Mr. Crane made sure to ask each presenter questions that helped the student to clarify and justify the work. He concluded the class by telling students that the problem could be solved in many different ways and now, when they solved a problem like this, they could pick the way they liked best because all the ways gave the same answer.

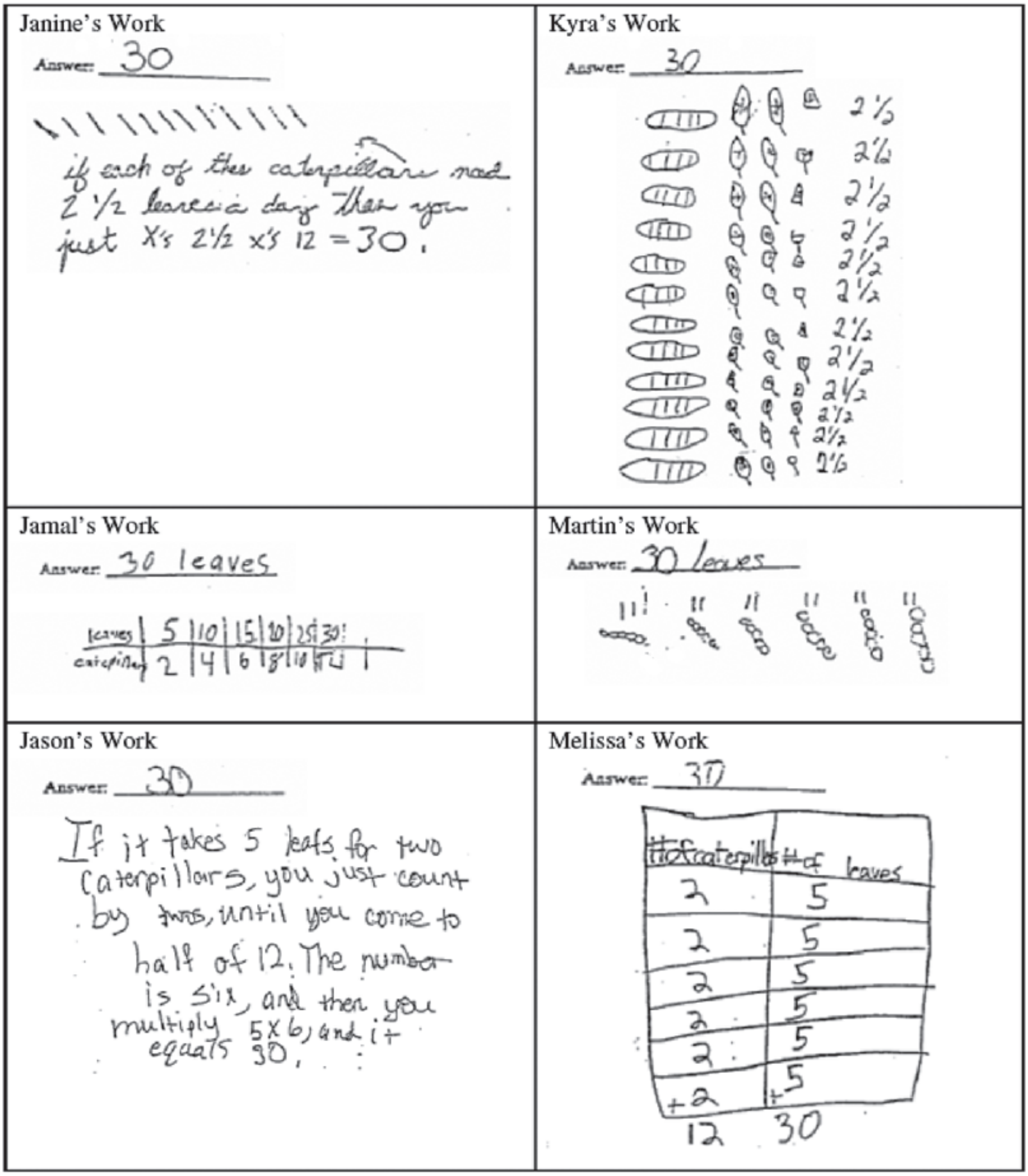

Fig. 0.2. Solutions shared by students in Mr. Crane's class

Analyzing the Case of David Crane

Some would consider Mr. Crane's lesson exemplary. Indeed, Mr. Crane did many things well, including allowing students to construct their own way of solving this cognitively challenging task and stressing the importance of students' being able to explain their reasoning. Students were working with partners and publicly sharing their solutions and strategies with their peers; their ideas appeared to be respected. All in all, students in Mr. Crane's class had the opportunity to become the "authors" of their own knowledge of mathematics.

However, a more critical eye might have noted that the string of presentations did not build toward important mathematical ideas. The upshot of the discussion appeared to be "the more ways of solving the problem, the better," but, in fact, Mr. Crane held each student accountable for knowing only one way to solve the problem. In addition, although Mr. Crane observed students as they worked, he did not appear to use this time to assess what students understood about proportional reasoning or to select particular students' work to feature in the whole-class discussion. Furthermore, he gathered no information regarding whether the two pairs of students who had gotten the wrong answer (Darnell and Marcus, and Missy and Kate) were helped by the student presentations of correct strategies. Had they diagnosed the faulty reasoning in their approaches?

In fact, we argue that much of the discussion in Mr. Crane's classroom was show-and-tell, in which students with correct answers each take turns sharing their solution strategies. The teacher did little filtering of the mathematical ideas that each strategy helped to illustrate, nor did he make any attempt to highlight those ideas. In addition, the teacher did not draw connections among different solution methods or tie them to important disciplinary methods or mathematical ideas. Finally, he gave no attention to weighing which strategies might be most useful, efficient, accurate, and so on, in particular circumstances. All were treated as equally good.

In short, providing students with cognitively demanding tasks with which to engage and then conducting show-and-tell discussions cannot be counted on to move an entire class forward mathematically. Indeed, this kind of practice has been criticized for creating classroom environments in which nearly complete control of the mathematical agenda is relinquished to students. Some teachers misperceived the appeal to honor students' thinking and reasoning as a call for a complete moratorium on teachers' shaping of the quality of students' mathematical thinking. As a result of the lack of guidance with respect to what teachers *could* do to encourage rigorous mathematical thinking and reasoning, many teachers were left feeling that they should avoid telling students anything.

A related criticism of inquiry-oriented lessons concerns the fragmented and often incoherent nature of the discuss-and-summarize phases of lessons. In these show-and-tells, as exemplified in David Crane's classroom, one student presentation would follow another with limited teacher (or student) commentary and no assistance with respect to drawing connections among the methods or tying them to widely shared disciplinary methods and concepts. The discussion offered no mathematical or other reason for students to necessarily listen to or try to understand the methods of their classmates. As illustrated in Mr. Crane's comment at the end of the class, students could simply "pick the way they liked best." This type of situation has led to an increasingly recognized dilemma associated with inquiry- and discovery-based approaches to teaching: the challenge of aligning students' developing ideas and methods with the disciplinary ideas that they ultimately are accountable for knowing.

In sum, David Crane did little to encourage accountability to the discipline of mathematics. How could he have more firmly supported student accountability without undermining student authority? The single most important thing that he could have done would be to have set a clear goal for what he wanted students to learn from the lesson. Without a learning objective in mind, the various solutions that were presented, although all correct, were scattered in the "mathematical landscape." If, however, he had targeted the learning goal of, for example, making sure that all students recognized that the relationship between caterpillars and leaves was multiplicative and not additive, he might have monitored students' work with this in mind. Whose work illustrated the multiplicative relationship particularly well? Did the students' work include examples of different ways of illustrating this relationship—examples that could connect with known mathematical strategies (e.g., unit rate, scaling up)? This assessment of student work would have allowed him to be more deliberate about which students he selected to present during the discussion phase. He might even have wanted to have the incorrect, additive solutions displayed so that students could recognize the faulty reasoning that underlie them. With an array of purposefully selected strategies presented, Mr. Crane would then be in a position to steer the discussion toward a more mathematically satisfying conclusion.

Conclusion

The Case of David Crane illustrates the need for guidance in shaping classroom discussions and maximizing their potential to extend students' thinking and connect it to important mathematical ideas. The chapters that follow offer this guidance by elaborating a practical framework, based on five doable instructional practices, for orchestrating and managing productive classroom discussions.