CPM EDUCATIONAL PROGRAM

CPM's 2023 Research Base
Executive Summary
Problem-Based Learning

This is a summary of CPM's 2023 research base on Problem-Based Learning. For more information and for references, please see the full report, available from https://cpm.org/research-base/.

What is Problem-Based Learning?

Researchers of mathematics education have largely accepted that problem solving is an essential part of doing mathematics; they now focus on how the distribution and delegation of power in mathematics classrooms influence students' opportunities to learn (Agarwal & Sengupta-Irving, 2019; Schoenfeld, 2016). While there are many types of authority at play in classrooms, the most relevant type of authority for mathematics learning is mathematical authority, or "who possesses mathematical knowledge that is taken as true" (Langer-Osuna, 2017, p. 238). This research base article focuses on the influence of mathematical authority on mathematics sensemaking in problem-based tasks.

CPM's Pillar of Problem-Based Learning refers to teaching that:

A. uses tasks that cannot be solved by a simple algorithm and that are often embedded in complex real-world contexts;
B. supports students to collaboratively construct knowledge through productive discourse practices (i.e., active, student-centered learning); and
C. shares mathematical authority with students.

In Problem-Based Learning, students construct knowledge by making connections to form big mathematical ideas.

Overview

Problem-Based Learning develops students' problem-solving skills, content knowledge, and eventually, the ability to be self-directed, independent learners (Duch et al., 2001; Hmelo-Silver & Barrows, 2015; Hammond, 2014). For example, empirical research has demonstrated that problem-based tasks are better than traditional tasks for supporting students to apply their knowledge to novel problems (i.e., transfer) and be self-directed in their learning (Hmelo, 1998; Hmelo & Lin, 2000; Schmidt et al., 1996). According to Langer-Osuna (2017) — a leading scholar on mathematical authority, agency, and identity — fostering shared authority between teachers and students supports increased student ownership of mathematical ideas (Bianchini, 1999; Ehrlich & Zack, 1997; Lotan, 1997), conceptual understanding (Hiebert et al., 1997), and positive identification with mathematics (Hand, 2012; Lotan, 2003).

CPM infers from this research that…

Problem-Based Learning supports students in learning mathematics in ways that will be useful to them in their future mathematics classes, in their careers, and in their lives outside of school and work. When students understand how to solve problems rather than how to correctly select and implement procedures, they develop mathematical authority and thus are more likely to enjoy and see the value in mathematics.

Why are problem-based tasks important for learning mathematics?

Over half a century of cognitive and sociocultural research on learners' mathematical thinking indicates that it is more effective (i.e., it better supports understanding and problem solving) although perhaps less efficient (i.e., it may not allow for "covering content" as quickly) to let students work with their peers to struggle through making sense of a problem and invent ways to solve it than it is to show students how to solve problems (Choppin, 2022; Schoenfeld, 2016). In fact, to do mathematics, the problem has to actually be a problem for the problem-solver (Schoenfeld, 2016). In sum, if we wish for students to develop productive mathematics problem-solving dispositions that follow them into adulthood, it is essential that students' own ideas become more central in instructional activities (Boaler & Greeno, 2000, Boaler & Selling, 2017).

CPM infers from this research that…

Problem-Based Learning supports students in authentically engaging in mathematics. Doing mathematics inherently requires struggle since problem solving only happens when the path to the solution is unclear. Problem-Based Learning is thus riskier for teachers than other forms of instruction because it requires putting students' ideas at the center of instruction—including their uncertainties and misconceptions—rather than the de facto correct ideas of the teacher and the textbook. Using heterogeneous grouping by randomly assigning students to teams can expose students to many different ideas, which then supports them in making decisions about which ideas to pursue. Though Problem-Based Learning may take more time than other ways of teaching, it pays off as students develop productive mathematics problem-solving dispositions that stick with them for life.

If Problem-Based Learning is important for mathematics learning, why is it not more widespread?

Unfortunately, many commercially available mathematics curricula today do not support problem solving (Larson, 2014). A 2022 study by Choppin et al. characterized the curricula of five mathematics textbook publishers — Math in Focus, Holt, Prentice Hall, Glencoe, CMP, and CPM — and found that all but CPM and CMP were substantially composed of tasks that served as delivery mechanisms for content rather than tasks that functioned as thinking devices that support student sensemaking.

CPM infers from this research that…

Problem-Based Learning is essential for supporting mathematical sensemaking, and CPM is one of the few secondary mathematics curricula that function as a “thinking device” for students rather than as a “delivery mechanism.” Because students do not come into the classroom as blank slates who will integrate everything they encounter exactly as the teacher and textbook intend, it is essential for curricula to support students in developing problem-solving skills as they persevere in making sense of mathematical tasks. Engaging in productive struggle during problem solving is much more aligned to the mathematical practices of mathematicians than is consuming a curriculum of discrete facts and skills derived from experts’ knowledge. In CPM materials, tasks are designed to engage students in mathematical practices that make learning the knowledge and skills found in traditional curricula much more sensible, enduring, and relevant.

Who is Problem-Based Learning good for?

Although it is common practice to argue that students with learning and intellectual disabilities need explicit instruction *instead* of problem-based learning, research has shown that these students can develop conceptual understandings through problem solving by (a) using mathematics tasks that are connected to real-world applications, (b) collaborating with peers and using manipulatives, and (c) inventing their own strategies rather than using standard algorithms (e.g., Lambert & Sugita, 2016). For more information, see the NCSM publication, Inclusion and Intervention: Understanding “Disability” in the Mathematics Classroom, by Jasien and Hayes (2022).

CPM infers from this research that…

Problem-Based Learning is appropriate for all students in inclusive classrooms.

What does it look like for students to engage in Problem-Based Learning?

Mathematical authority is not a binary (e.g., students do not either have or do not have mathematical authority), but rather occurs along a spectrum, and is flexible (rather than static), expanding (rather than finite), and situational (rather than stable across time)(Bishop et al., 2022). When students have a high degree of mathematical authority, they “are ‘authorized’ to solve mathematical problems for themselves, are publicly credited as the ‘authors’ of their ideas, and develop into local ‘authorities’ in the discipline” (Stein et al., 2008, p. 332).

CPM infers from this research that…

Problem-Based Learning requires inviting students to make mathematical decisions and contribute substantive mathematical ideas. In other words, in a classroom where Problem-Based Learning is flourishing, students exercise a high level of mathematical authority by making conjectures, explaining their work and justifying their reasoning to one another, and building on each other’s ideas. Of course, this is unlikely to happen at the beginning of the school year in any classroom because it requires building a classroom culture that supports students in taking on more and more mathematical authority over time. Mathematical authority can shift from moment to moment and day to day depending on student engagement and the mathematical ideas of the lesson, but over time, teaching for Problem-Based Learning should support all students in exercising mathematical authority.

How is Problem-Based Learning different from more traditional mathematics learning?

In too many classrooms, the teacher and textbook are the primary sources of claims about what is right and how mathematical thinking should unfold (Engle & Conant, 2002; Herbel-Eisenmann & Wagner, 2007; Litke, 2015, 2020; Mehan, 1979; NCTM, 2014). In classrooms where the teacher and textbook are the primary sources of mathematical authority, students' mathematical sensemaking is backgrounded in favor of behavioral compliance, with demands like giving the teacher complete attention and always being on task without socializing (Cazden, 2001; Hand, 2012). Teaching in ways that support students to become problem solvers who enjoy mathematics requires making space for students to be their full selves, including by attending to students' relationships with each other (e.g., their social and academic status), their emotional and physical needs, and allowing goofiness and social talk to occur with mathematical discourse (Joseph, 2021).

For many students — including high-achieving students — traditional classroom practices that do not make room for problem-solving activities like exploration and justification can lead students to disaffiliate with mathematics (e.g., the common phrase, "I'm not a math person;" Boaler & Greeno, 2000; Pope, 2001).

CPM infers from this research that…

Problem-Based Learning may require teachers to significantly shift their pedagogy, moving from a teacher-centered approach toward a student-centered approach. While traditional teaching methods may work for some students in terms of performing well on assessments, they do not work well for most students. Even more, there are too many students with good grades or in advanced mathematics classes who achieve in mathematics not because they enjoy or see intrinsic value in it, but because they know how to "do school" and see its importance for their future (e.g., collegiate gatekeeping). Problem-Based Learning invites more students to see themselves as belonging in mathematics class and creates more meaningful mathematics learning for those who already see themselves as belonging.

Does facilitating Problem-Based Learning mean that I no longer take on the role of being a mathematical authority?

Sharing mathematical authority does not mean that teachers should withdraw or abdicate their own authority. The idea that authority is finite (i.e., there is only so much to go around) is due to the widespread misconception that mathematical authority is a zero-sum game (Bishop et al., 2022). When teaching for Problem-Based Learning, teachers act as expert learners by using open questions to scaffold students toward mathematical understandings, thus offering students a kind of apprenticeship in mathematical thinking (Hmelo-Silver & Barrows, 2015).

CPM infers from this research that…

Problem-Based Learning expands the mathematical authority within a classroom as teachers share, rather than give away, their intellectual authority with students. In fact, when teachers abdicate their mathematical authority, students' mathematical reasoning can suffer. By modeling the practices of expert learners rather than acting as a source of finished knowledge, teachers support students in reasoning mathematically.

How can I share my mathematical authority as a teacher without short-changing the value of Problem-Based Learning?

Teachers can support students to develop mathematical authority by eliciting and probing student thinking (Arnesen & Rø, 2022; Ellis et al., 2019; Hamm & Perry, 2002), building on students' ideas (Arnesen & Rø, 2022; Drageset, 2014; Sherin, 2002); supporting students to engage in evaluative work by making connections (Arnesen & Rø, 2022; Drageset, 2014; Kazemi & Hintz, 2014; Stein et al., 2008), assigning competence (Esmonde, 2009; Hand, 2012; Jilk, 2016), and facilitating peer-to-peer mathematical relationships (Langer-Osuna, 2015).

CPM infers from this research that…

Problem-Based Learning requires teachers to exercise their mathematical authority while still sharing mathematical authority with students. Teachers can do this by questioning in ways that:

- probe process before probing content (e.g., encouraging the student(s) to explain their work and justify their reasoning to one another, or using study team and teaching strategies to help students learn from each other);
- model mathematical practices (e.g., asking for detailed explanations (how) and justification (why), or asking for connections between multiple students' thinking and between multiple representations); and
- foster mathematical discourse amongst students (e.g., asking open-ended questions that make space for students to share and build on each other's ideas, strategies, representations, and explanations).

What student moves influence mathematical authority in classrooms that feature Problem-Based Learning?

Langer-Osuna et al. identified a small set of student actions that frequently preceded shifts in who exercised mathematical authority in teams, with all of these actions involving students publicly naming aspects of their teamwork: asking for clarifications (e.g., "So what are we working on?"), stating the work to be done (e.g., "We are supposed to get a new card."), and making their own process the object of consideration (e.g., by counting manipulatives aloud).

CPM infers from this research that…

Problem-Based Learning reflexively supports and requires students to develop a growth mindset, persistence with an expectation of difficulty during mathematical problem solving, and willingness to take social and academic risks, all essential aspects of developing mathematical authority. For example, students engaging in Problem-Based Learning will often need to ask for clarifications from their teammates, direct their team members, find resources for problem solving, and expose their own uncertainties. CPM materials are designed to support students in engaging in these activities and more through the incorporation of team roles, which are used to their fullest potential when they support students in making their mathematical ideas (including their uncertainties) public.