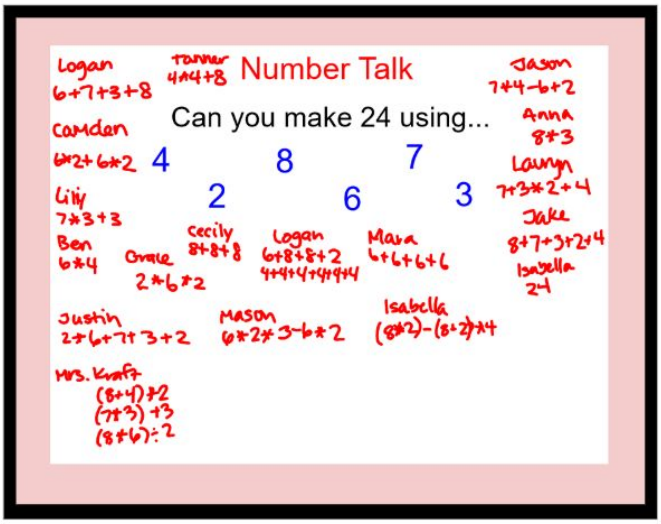
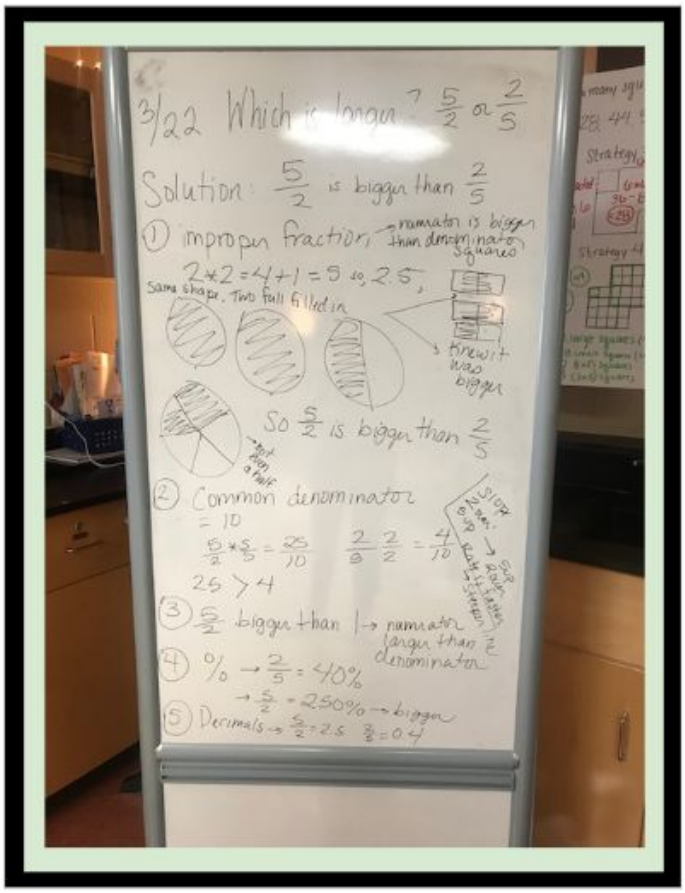
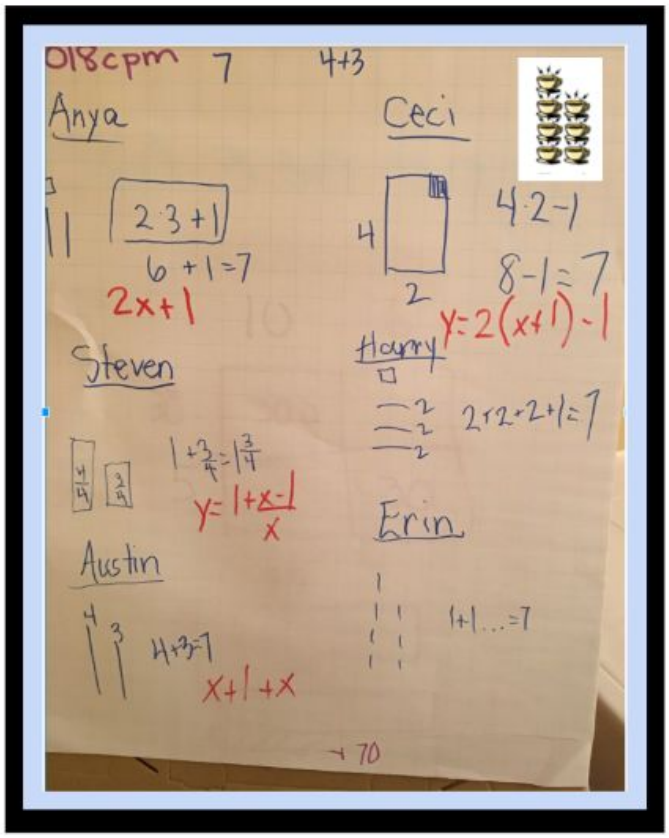


Making Student Thinking Visible with Number Talks





By

Angela Kraft and Pam Lindemer

The following paper captures the results of a multiple-year investigation of number talks conducted by the CPM Educational Program Teaching Redesign Corps (http://cpm.org/trc). Our investigations began in July of 2016 as a set of research proposals crafted during the TRC 3.0 meeting in Las Vegas, Nevada. Since that initial meeting, several cohorts of Teacher Researchers have collaborated to explore the instructional practice of conducting number talks by systematically changing our teaching behavior while observing and recording differences in student learning. We, the lead authors, would like to acknowledge the exceptional effort made by our fellow researchers Leah Callister, Dan Chandler, Mark Jones, Tony Jones, Karen Kurcz, Megann Line, Megan Mastrocola, Chad Ophime, and Erin Treu. Our collective student populations are diverse, with representation from multiple SES levels, rural, urban, suburban, large and small schools, grades 6 -12, and courses including pre-algebra through calculus. We have synthesized what we feel are the best practices and activities that were effective in all of our classrooms.

Disclaimer: CPM is glad to share the findings from the TRC investigations, but these are just ideas. It is expected that teachers will rely on their knowledge of their students, the mathematics they are teaching, and the circumstances surrounding their specific teaching assignment when modifying their own behavior and selecting appropriate instructional strategies. A given strategy may have a positive effect on student learning in some situations and a negative effect in others. Please use your own best judgement as you continue to improve your teaching practice.

Table of Contents

1. What is a Number Talk?

Have you ever wondered what your students were thinking? Do you have students who provide you with the correct answer and you are still not convinced that they understand the underlying mathematical concepts and procedures? Do they fight you to show their work? We often found ourselves monitoring our students work in teams but the evidence of their thinking on paper left us with questions. We needed something to help tackle these problems early in our careers. These classroom issues helped us land upon number talks! We found that number talks provide teachers with an opportunity to get inside the student's head and clearly understand their thinking. Try this number talk: mentally Solve 18 * 5.

What did you get?
How did you get that answer?
Can you precisely explain your thinking?
Can you solve this problem in a different way?
Can you draw a diagram to illustrate your thinking?

We use these types of questions daily when working with teams of students. We are constantly pushing students for multiple solutions. Our ultimate goal is for students to be able to explain their reasoning of solving problems so that others around them can understand their thinking processes. We recognized this was a huge need in our math classrooms; a tough skill that needs to be modeled appropriately.

Pam teaches Math Lab to a group of disaffected students. These students have a history of having difficulty in mathematics and are below grade level when entering Algebra 1 in the 9th grade. It is a tough crowd and engaging these students and others like them is difficult. In an attempt to reach them, she has introduced number talks into their daily routine. Gavin is a new student in class and so Pam reminds her students of their norms.

Pam: All students have important ideas to share and I'm interested in hearing your thinking.

Gavin: (Mutters something under his breath.)

Pam: Gavin, are you willing to share your thoughts?

Gavin: That's different. Teachers are usually telling me what to think.

How many times have our students felt that way? Obviously Gavin needed convincing. Over time, he became an active participant in number talks, regularly sharing his ideas with the class. Pam's curiosity and determination to understand Gavin's thinking convinced him that not only were his ideas interesting, but they were important to the learning process.

An Illustration

Angela and her seventh grade students were working on building number sense. As part of this work, Angela wanted her students to recognize that there are a variety of ways to solve problems. She chose the problem 18 * 5. After asking the students to put all materials away and clear off desks, no pencils or paper, she projected the problem 18 * 5 on the board. Students were asked to mentally solve the problem and give a thumbs up on their chest once they arrived at a solution. To allow time for more students to finish, she asked those with thumbs up to try to solve the problem in more than one way and identify how many different strategies were used by holding up fingers. She provided enough wait time until most students had a thumbs up. To begin the class discussion, she asked for volunteers to share solutions and she recorded them under the projected problem (even if solutions were incorrect). Next, she asked for any volunteers to share their thinking on how they arrived at their solution.

Angela: Does anyone want to volunteer to explain how they arrived at their solution? I love to see the many hands raised to share! How about we start with Quinn. Quinn, which answer will you be defending?

Quinn: 90. I first looked at it and knew I could multiply easily with 10.

Angela: Hang on, what do you mean by "it"?

Quinn: Oh, right. I mean I looked at 18 * 5 and realized I can change 5 to 10 and multiply more quickly. So I was know that 18 * 10 is 180. Then I knew I had to change my answer to 90 because I had to take half of 10.

Angela: Great, thank you Quinn. Does anyone have an idea of what we could name Quinn's method? Ashley.

Ashley: I see that Quinn doubled the 5 to get 10, so maybe we could call it the doubling method?

Angela: Nice, Ashley. I like that. Can everyone see what Ashley is saying? Great. Did anyone solve this is a different way that would like to volunteer to share? Brady. Which solution will you be defending?

Brady: 90. I am really good with math facts, so I decided to change 18 to 9 and multiplied 9 * 5 to get 45. Then I added two 45s together to get 90.

Angela: Brady, did I record this as you intended?

Brady: Yes.

Angela: Great, thank you Brady. What can we name Brady's strategy? Jen.

Jen: Maybe we could call it halving?

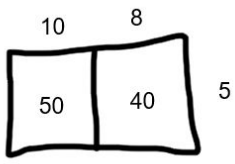
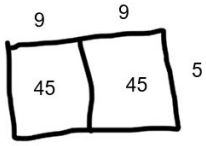
Angela: Interesting Jen. Why do you think that?

Jen: Because I see that he took half of 18 to get 9.

Angela: Class, what do you think? Agree with Jen?

Angela carefully continued this process recording each volunteer's response exactly as they shared. Angela asked clarifying questions and prompted the students for more details to help model appropriate and precise communication strategies. Each time a volunteer was willing to share, Angela asked the student to indicate which recorded solution the student was going to defend. She then asked the class to either name the strategy after the student who volunteered or named the appropriate mathematical vocabulary or property the student used. The following chart shows the variety of solutions her seventh grade classroom came up with for 18 * 5.

Doubling	**Halving**	**Friendly Number/ Round and Adjust**	**Conventional Algorithm**
18 * 10 = 180 180/2 = 90	9 * 5 = 45 45 * 2 = 90	20 * 5 = 100 2 * 5 = 10 100 - 10 = 90	$\begin{array}{r} {\scriptstyle 4} \\ 18 \\ \times\ 5 \\ \hline 90 \end{array}$

Array Model	**Repeated Addition**	**Repeated Multiplication**	**Halving**
 50 + 40 = 90	18 18 18 18 + 18 90	18 * 2 = 36 18 * 2 = 36 18 * 1 = 18 36 + 36 = 72 72 + 18 = 90	18/2 = 9 9 * 5 = 45 9 * 5 = 45 45 + 45 = 90
Repeated Addition	**Count Up**	**Count Up**	**Distributive Property**
5+5 = 10 5+5 = 10 5+5 = 10 5+5 = 10 5+5 = 10 5+5 = 10 5+5 = 10 5+5 = 10 5+5 = 10 Nine 5s + Nine 5s = 90	18, 36, 54, 72, 90 Five 18s = 90.	5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 Eighteen 5s is 90	10 * 5 = 50 8 * 5 = 40 50 + 40 = 90
Array Model		**Incorrect Conventional Algorithm**	
 45 + 45 = 90		18 * 5 140	

After all student strategies were shared, Angela asked a few follow-up questions to close out the number talk.

Angela: Look at how many different ways we all solved this problem! Amazing, right!? Let's take a look at the first two strategies shared. Quinn's doubling and Brady's halving. How are these two strategies alike or how are they different? Henry.

Henry: I see that they both took a number and changed one to make the

problem simpler.

Angela: Nice observation, Henry! Any others? Susan.

Susan: I also noticed that both Quinn and Brady changed the numbers in the beginning and remembered to adjust the answer in the end.

Angela: Yes, I agree. They attended to precision, MP6!!! Does anyone see any differences? Joe.

Joe: One doubled the second number and the other halved the first number.

Angela: Excellent!

We learned that for number talks to be successful, students need to feel comfortable sharing strategies whether they know their solutions are correct or not. They must also feel secure knowing they will not be judged. We had to spend the first month of school building a classroom culture where mistakes were valued and students recognized the importance of learning from their mistakes. We read Carol Dweck's work on growth mindset (2006) and attended workshops with Jo Boaler who states, "When I am working with children and they say something that is incorrect, I consider their thinking with them and see this as an important opportunity for learning. When students know that I am not judging them harshly and that I genuinely value errors, they are able to think more productively and learn more." (Boaler, 2015)

How We Started

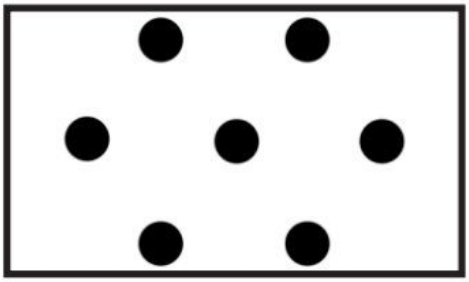
In order to implement effective number talks, we found it essential to establish a growth mindset culture during the first weeks of school. We each did this in a variety of ways, through videos, music clips, activities, or number talks themselves. We came to understand that growth mindset is a journey, not a destination, and we had to review and revisit this idea often throughout the school year. We instituted Mindset Mondays to continue to remind our students to keep a growth mindset. Additionally, number talks allowed us as teachers to model how to ask clarifying questions and prompt students without implying negativity or evaluation.

Next, we found it important to establish a common protocol for conducting number talks. The entire research team helped establish the routine outlined below, noting that the students grew very comfortable when a predictable format was maintained.

Number Talk Protocol

1. Students are instructed to not use paper or pencils, and to place a fist on their chest to show they are ready to begin.
2. The teacher displays the problem on the board and allows think time.
3. Students mentally solve the problem and put a thumb up when the have a solution to share.
4. Students who solve the problem one way are asked to think of additional ways to solve the problem while they wait for their peers.
5. When most thumbs are up, the teacher asks for volunteers to share answers while the teacher records on the board.
6. When there seem to be no more answers, the teacher asks students to quietly identify an answer they are willing to defend. In turn, each volunteer picks an answer and offers an explanation of how they arrived at an answer. Teacher records exactly what is said. There may be more than one solution offered for a given answer.
7. The teacher continues to ask clarifying questions or prompts them for more information to help students communicate their thinking.
8. Be mindful of time, generally 10-12 minutes per talk.

Researchers who were most successful started with dot talks or visual patterns and committed to using the routine frequently during the first few weeks of school. Cathy Humphreys and Ruth Parker suggest this is a great way to introduce number talks in their book, *Making Number Talks Matter*. Some researchers did them every day for ten days, others committed to doing them 10 times in the first month of school. During these talks we focused on creating routines and establishing norms around the procedure. The more time we spent on the first few dot talks, the better the future talks became. For example, Pam began with the dot image at the left. She asked students to indicate how many dots they see without actually counting and then followed the



same protocol as above asking students to provide a thumbs up when they are ready to explain how they arrived at the number of dots. She prompted students to think of multiple solutions. A variety of these types of dot talks can be found using Google.

It is also important to point out that students will make mistakes when sharing solutions or strategies. In connecting with growth mindset, often times great learning can come out of finding where misconceptions exist. Sometimes these conversations can be extremely powerful. Remember to celebrate these mistakes and allow the student to talk through them. Teachers can follow mistakes up by asking if anyone would want to revise their answer and share their thinking; or follow up with, “Does anyone know how someone might have arrived at this solution?”

Purpose of Number Talks

Early in the year, Pam chose a string of addition problems for her number talk, hoping that students would share multiple strategies and be exposed to some new ideas that they might make their own. She began with the problem 99 + 45. Students shared answers and then she asked for a volunteer to defend their answer.

Kyus: I got 144. I lined up the two numbers in my head and did 9 plus 5 is 14. You write down the 5 and carry the 1. Then I did 9 plus 4 and got 13. I add the 1 and got 14. That gives 144.

Pam: Like this?

$$\begin{array}{r} 99 \\ +45 \\ \hline 144 \end{array}$$

Kyus: Yup.

Pam: Did anyone else do it the way Kyus did? (Several students raise their hands.) Did anyone solve the problem in a different way?

Remi: I got 144 just by adding 100 and 44.

Pam: Where did the 100 come from?

Remi: I borrowed 1 from 45 and added it to 99 to get 100. That left 44. When you add 100 and 44, you get 144.

Kyus: Remi! That's so easy it's genius!

Remi: (Straightening in her seat and smiling, said nothing.)

Pam: Did anyone else use Remi's strategy? (Only one student raised their hand.)

Kyus: Does it always work?

Pam: What do you think? Does it always work? (Remi thought it did, but was the only one to say so.)

Pam: Let's try another problem and test Remi's strategy.

Pam continued the number talk using the problems 99+29, 999+79, and 999+357. The next day there was a note posted on the classroom wall titled *Remi's Strategy* with three examples and the number talk involved. The author added three more problems that could be solved by using Remi's Strategy. This organic, unprompted act was a clear indication that some of the students were beginning to use this give and take strategy and make it their own.

Number talks will be most successful when the teacher has a purpose in mind, and clearly communicates that purpose with students ahead of time. Number talks allow the opportunity for teachers to model precise mathematical vocabulary and high-level mathematical discourse. We found that the learning benefits resulting from the consistent implementation of number talks transferred to group work; students were spontaneously using more mathematically appropriate vocabulary and asking appropriate clarifying questions of each other. Why do number talks? We noted six good reasons:

<u>Benefits of Number Talks</u>

- Emphasizes content
    - Explore multiple ways to a solution
    - Preview, review and spiral content
- Improves discourse
    - Explain and critique mathematical thinking/reasoning
    - Use, hear, and expand mathematical language/vocabulary

- Increases accessibility of mathematical ideas
- Strengthens number flexibility and numeracy
- Encourages use of academic vocabulary
- Addresses equity and inclusion issues

2. Math Talks

When doing number talks, we learned that not all problems are created equal. Experience taught us something important about the problems that lend themselves well to number talk discussions. First, they had to have a low floor and high ceiling. These problems need to be easily accessible to all students regardless of their mathematical background and allow for students to extend their thinking. Second, successful problems lend themselves to multiple paths to a solution. In addition, we found that student engagement increased when students were offered a variety of types of number talks. Once we realized the many options available for number talks, we determined it would be more fitting to call these math talks. We've listed all the different types of math talks implemented by the research team below. See the appendix for more information about each.

<u>Math Talks</u>

Visual
- Pattern talks
- Dot talks
- Splat!
- Cube conversations

Reasoning
- Which one doesn't belong? (WODB)
- Estimation clipboard
- Can you make it?

Numercial
- Number talk
    - Basic operations
    - Fraction/decimal/percent
    - Four 4s
    - Which is greater?
    - Exponents

- Trigonometry
- Sequences
- Team Number Talk
- Guest recorder

What Teachers and Students are Doing During Math Talks

Teacher	Students
● Engaging in all NCTM Math Teaching Practices ● Purpose - make the goal of math talk clear before beginning ● Communicate norms - remind students of expectations during math talks ● Recording student thinking ● Allowing sufficient wait time for all students to formulate thoughts about the problem ● Anticipate student responses - be ready to share strategies students might not be able to come up with ● Revoicing - after a student shares a strategy, teacher calls on another student to explain that strategy in their own words ● Rough Draft Talk - allowing individual think time and providing an opportunity for students to share their ideas among a small group before the large class share ● Volunteer skeptic - ask if a volunteer would like to act as a skeptic of a particular strategy ● Strategy posters - create an anchor chart displaying the strategies used throughout the math talks ● Naming strategies ● Clarifying questions - ask the class	● Engaging in all Common Core Math Practice Standards ● Use sentence starters to initiate conversation: ○ I agree/disagree with you because… ○ What I heard you say was… ○ What were you thinking here… ○ What did you start with… ○ Why did you choose that operation… ○ I still have questions about… ○ This reminds me of… ● Listening for understanding ● Making sense of another person's strategy ● Using precise language and vocabulary when sharing strategies ● Exhibiting a growth mindset

if anyone has any questions from the strategies shared ● Exhibiting a growth mindset ● Prompting students for more explanation and precise language and mathematical vocabulary ● Closure - offer closure to the math talk connecting back to the purpose	

Teachers

NCTM suggests effective teachers provide students opportunities to engage in high-level thinking by regularly implementing tasks that promote reasoning and problem solving and build procedural fluency from conceptual understanding. Math talks provide a structure where teachers can engage in both of these practices. In Angela's illustration at the beginning, she was able to model reasoning and problem solving by encouraging students to find multiple strategies and share their solutions. In addition, Angela promoted procedural fluency by exposing students to strategies they may not have considered themselves.

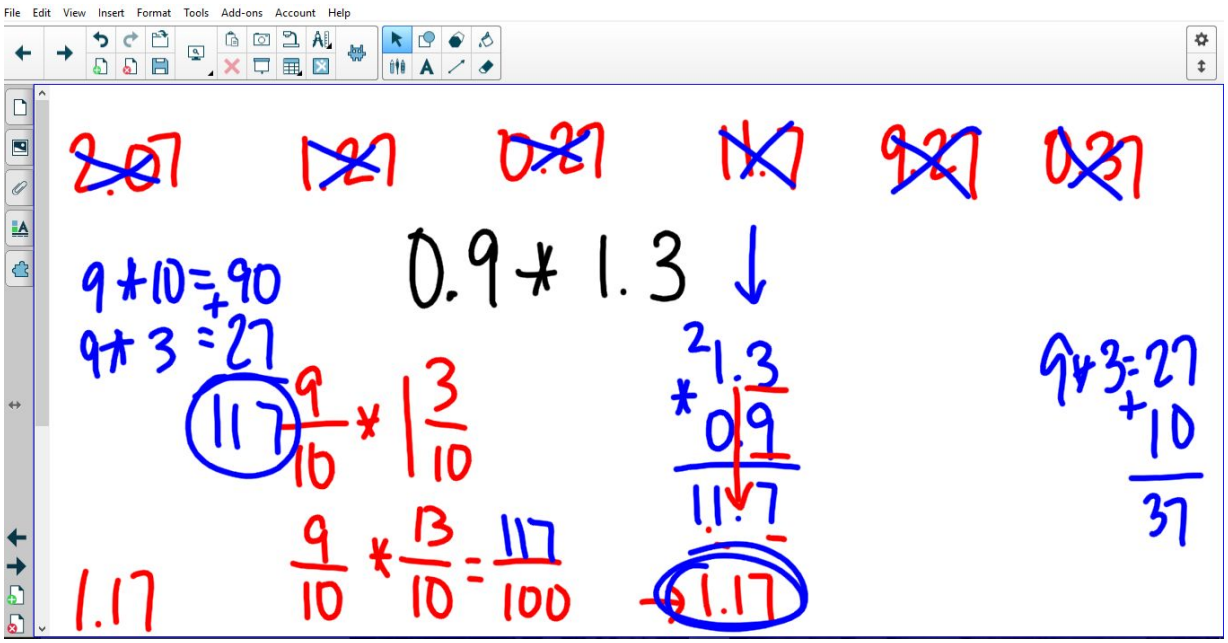
Students

NCTM suggests that learners should have experiences that enable them to construct knowledge socially, through discourse, activity, and interaction related to meaningful problems. Math talks allow students to listen and construct viable arguments if they agree or disagree with a volunteer's strategy. In addition, students need to learn how to effectively communicate their strategy through precise mathematical vocabulary and language; enough so that all others can understand their thinking. In Angela's illustration, she modeled how to ask appropriate clarifying questions when students were stuck in sharing a solution and prompted students to use precise vocabulary when sharing.

3. What Happens When the Math Talk Doesn't Go As Planned?

In our experience, some math talks worked out better than others. Sometimes almost all students were engaged, and many made contributions; other times only a few. It is important to reflect on the math talks that did not go well to determine why and adjust in the future. We worked on our own mindset to learn from our mistakes, and NOT GIVE UP! When time is an issue, make attempts to connect the math talk to the day's lesson. Being able to connect a math talk to the learning goal for a lesson helps justify spending time on the routine. Additionally, connecting math talks to common mistakes made on assessments is another great tie to the current classroom material and will help alleviate common misconceptions in a unique fashion. Effective math talks do not need to take much time. Ten minutes, maximum is about right.

For example, the following image shows a number talk that did not go as intended. The crossed out answers at the top indicate all of the initial solutions shared. You will notice that none of the shared solutions were correct. Angela knew this was the case, but moved onward asking for any volunteers to share strategies in the hopes that maybe through the sharing process, students will flounder upon the mistake.
After a volunteer shared his solution of 11.7, Angela posed the next question.



Angela: Gosh, I wonder how we know if our answer is correct? Is there another way we could tackle this problem to check the work?

Jack: Well, we could convert these decimals to fractions.

Angela: Great idea Jack, let's see what happens. We know that 0.9 is what

as a fraction?

Lilly: Nine tenths.

Angela: Yes, and 1.3?

Lilly: One and three tenths.

Angela: Great. Now let's multiply them. Can anyone walk us through this process? Ben.

Ben: We first need to convert one and three tenths into an improper fraction by multiplying the bottom number by the whole number and adding the top number to get thirteen tenths.

Angela: Great Ben! Is there more precise mathematical vocabulary Ben could have used to describe the top and bottom of a fraction?

Haley: Yes, numerator for top and denominator for bottom.

Angela: Nice! Ok, now let's multiply across. 9 * 13 is 117 and 10 * 10 is 100. That makes 117/100ths. Is 11.7 the same as 117/100ths?

Jack: No.

Angela: A-ha! So do we need to make any changes?

Jack: Yes, we need to move the decimal point one place to the left.

Angela: Great! Look at this! Let's go back to our original strategy. Where was our mistake?

This type of math talk may happen for you. Do not get discouraged. There is a lot of learning that can come out of mistakes. It would be wise to follow this math talk up with a similar problem in the near future to see if these strategies begin to stick with your students.

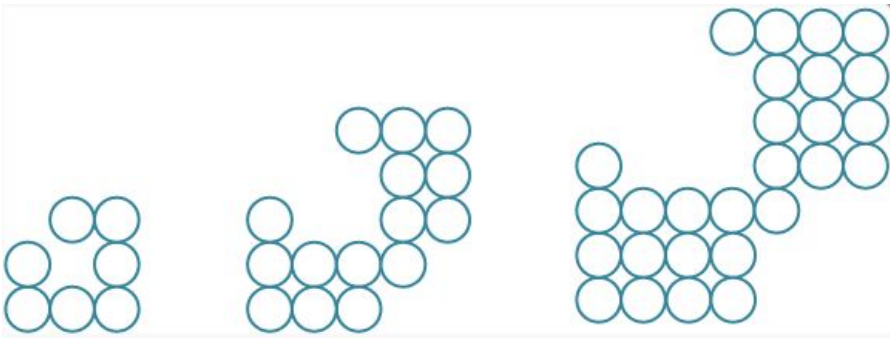
4. Now That We Can See What Our Students Are Thinking

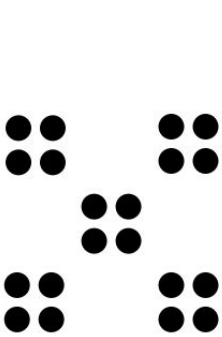
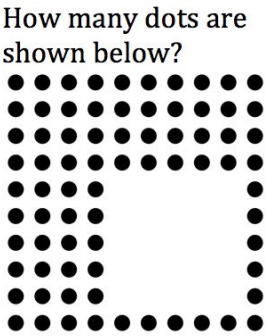
In the general course of teaching math we all come across a situation where a student understands what mathematical process needs to be applied, but struggles in the attempt to explain why. For us, math talks have become an efficient method for working on this skill in the secondary classroom. So, whether our students were learning pre-algebra or calculus, math talks proved to be a transformational strategy for eliciting the mathematical teaching practices as well as the common core math practice standards for both teachers and students. We found that establishing a safe and positive class culture was of critical importance. Our students needed to learn that they were very capable of recognizing patterns and finding multiple ways to solve a problem, and we needed to remember that any change in our teaching practices would require time, patience and perseverance.

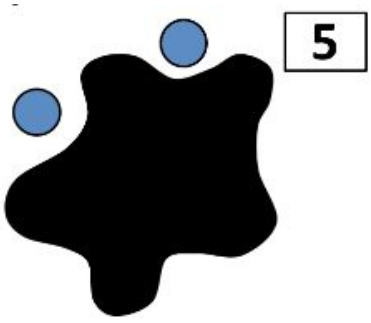
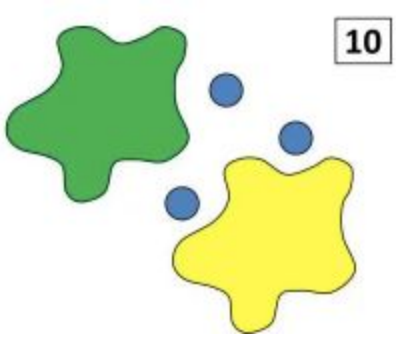
Performing math talks regularly with your students will help develop number sense, increase confidence in solving mental calculations, improve student justifications and explanations using precise mathematical language and vocabulary, and expose students to the multitude of strategies that exist to solve problems. They also serve as fun tools to keep your students on their toes, offering a quick brain break or warm up to any lesson.

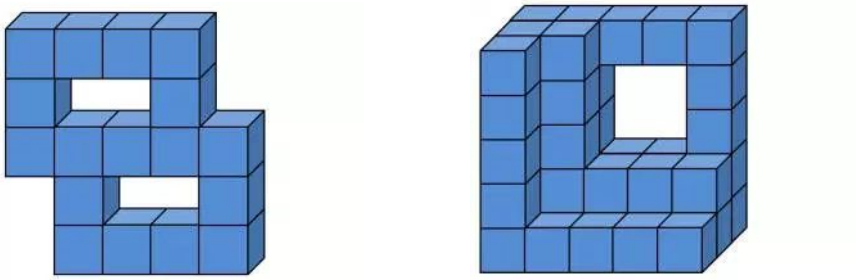
5. Appendix

Visual

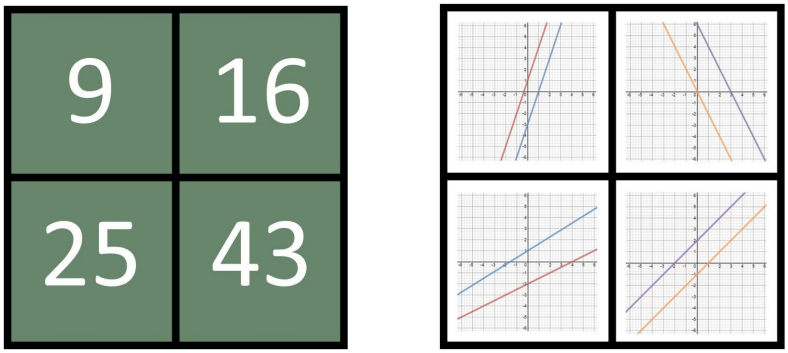
Pattern Talks	Grade Level: 6-12
Nguyen, F. (2013). *Visual Patterns*. Retrieved from http://www.visualpatterns.org/contact.html VanDerWerf, S. (2017, June 27). Secondary Number Talks (I'll Convince You With Ducks) [Web log post]. Retrieved from https://saravanderwerf.com/2016/06/27/secondary-number-talks-ill-convince-you-with-ducks/	
Description: Without counting each circle individually, how many circles do you see?	
	
Extension: If this were the second figure in a pattern, how could you write an expression to represent the total number of circles? Can you write an expression for each of the visual representations shared?	
	

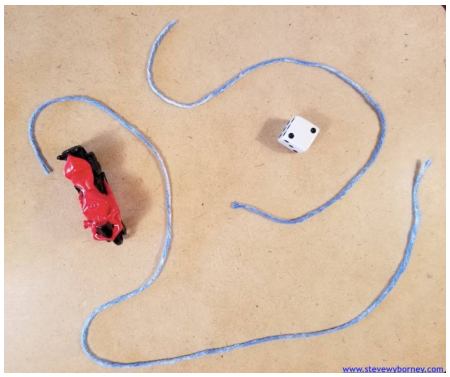
Dot Talks	Grade Level: K-12
"Jo Teaching a Visual Dot Card Number Talk." *YouCubed*, www.youcubed.org/resources/jo-teaching-visual-dot-card-number-talk/.	
Description: Without counting each dot, how many dots do you see?	
 	
Extension: Consider removing the image from display after a few seconds to force students to focus on active listening rather than trying to determine more strategies as students begin to share.	

Splat!	Grade Level: 6-12
Wyborney, S. (2017, August 18). Steve Wyborney's Blog: I'm on a Learning Mission. Retrieved from www.stevewyborney.com	
Description: How many dots are under the Splat? Different colored Splats cover different numbers of dots.	
 	

Cube Conversation	Grade Level: K-12
Wyborney, S. (2017, August 18). Steve Wyborney's Blog: I'm on a Learning Mission. Retrieved from www.stevewyborney.com	
Description: Without counting, how many cubes do you see?	
	

Reasoning

Which One Doesn't Belong	Grade Level: K-8
Danielson, Christopher, et al. "Which One Doesn't Belong?" Retrieved from http://wodb.ca/	
Description: Determine which one of the set does not belong. Be ready to explain why.	
	
Extension: Students can create these to share.	

Estimation Clipboard	Grade Level: K-12
Wyborney, S. (2017, August 18). Steve Wyborney's Blog: I'm on a Learning Mission. Retrieved from www.stevewyborney.com	
Description: What is the length of each piece of yarn in whole inches?	
	

Can You Make It?	Grade Level: K-8
Fun Games 4 Learning, Teresa. "Math to Make Them Think!" *Fun Games 4 Learning*, 1 Jan. 1970, fungames4learning.blogspot.com/2014/07/math-to-make-them-think.html.	
Description: Can you make 24 using any operations with the numbers 2, 4, 6, 7 or 8?	
Extension: Create your own!	

Numerical

Number Talks	Grade Level: K-12
Nguyen, F. (2013). NT 5-8. In *Math Talks*. Retrieved from http://www.mathtalks.net/nt-5-8.html Parish, S. (2014). *Number Talks: Mental Math and Computation Strategies*. Sausalito, CA: Math Solutions.	
Description: Mentally solve this problem.	
Basic operations: 99 + 103 | 200 - 22 | 16 x 9 | 101 / 4	

Fraction/decimal/percent: 1 / (2/3) | .6 x .2 | 8% of 10
Four 4s: Use four 4s to make 0 using only (+ - x /). 0 = 4 ÷ 4 × 4 − 4 = 44 − 44 https://en.wikipedia.org/wiki/Four_fours
Which is greater? 3/16 or 5/20
Exponents: $4^3 / 4^2$
Trigonometry: sin θ = -1

Sequences	Grade Level: 9-12
Pam, Harris. "Problem Strings." *Pam Harris Consulting*, 19 Jan. 2018, http://www.mathisfigureoutable.com/problem-strings/	
Description: What are the next two terms in the sequence?	
4, 2, … 1, 5, … 3.2, 6.4, … 0.1, 0.01, ...	
Extension: Find the next two terms in the sequence if it were both arithmetic and geometric. What if it were neither?	

Team Number Talk	Grade Level: K-12
Description: Perform any type of math talk with a team of 3-4 students. This allows students the opportunity to share with a smaller group before sharing their ideas with the whole class and can help increase participation. Example responsibilities for each team role: Facilitator introduces the number talk, writes down the solutions shared among the group, and encourages all team members to share. Task Manager encourages all members to share and ensures solutions are shared one at a time. Recorder/Reporter records solutions and strategies shared. Resource Manager grabs all materials needed (whiteboard and marker) and records for the recorder/reporter when he/she shares a solution/strategy.	

Guest Recorder	Grade Level: K-12
Description: Instead of having the teacher act as the recorder of the math talk, ask a student volunteer to record. This might take a bit more time to complete the math talk, however has shown benefits in forcing students to use more precise language when sharing strategies as often times students might not have the same mathematical background as a teacher and might to be able to “save” the student who is trying to explain a strategy.	

6. References

Boaler, Jo. *What's Math Got to Do with It?: How Teachers and Parents Can Transform Mathematics Learning and Inspire Success*. Penguin Books, 2015.

Dweck, Carol S. *Mindset*. Robinson, an Imprint of Little, Brown Book Group, 2017.

Humphreys, Cathy, and Ruth E. Parker. *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding*, Grades 4-10. Stenhouse Publ., 2015.

Jansen, Amanda, et al. “Rough-Draft Talk in Mathematics Classrooms.” *Mathematics Teaching in the Middle School*, vol. 22, no. 5, 2017, p. 304., doi:10.5951/mathteacmiddscho.22.5.0304.

Leinwand, Steve. *Principles to Actions: Ensuring Mathematical Success for All*. National Council of Teachers of Mathematics, 2014.