

## Developing a Positive Mathematical Identity

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In mathematics classrooms, mathematical identity is mediated by how students engage with mathematics. Consequently, mathematics teaching involves not only helping students learn concepts and develop skills and understanding but also empowering students to see themselves as capable of participating in and being doers of mathematics. In the Bike and Truck vignette above, Ms. Shackelford orchestrated a mathematical discussion with her students to engage in practices allowing them to see themselves as doers of mathematics. In doing so, Ms. Shackelford positioned her students as mathematically competent.

In too many high school mathematics classrooms, students are passively engaged with mathematics, and little mathematical discourse occurs. Reasoning and sense making are rarely encouraged, and mathematics is positioned as having little relevance to students' lives or experiences. In other classrooms, students are active participants, engaging in reasoning and sense making, striving to make their mathematical thinking visible and intelligible to others, using multiple forms of discourse, and critiquing their world through the use of mathematics.

Depending on the context, one's mathematical identity reflects a sense of oneself as a competent performer, able to do mathematics. How students are positioned to participate in mathematics affects not only what they learn but also how they come to see themselves as learners. The ways in which students view themselves as learners of mathematics greatly influence how they participate (Bishop 2012; Nasir and Hand 2006). Developing students' identities should be part of teachers' daily work, in which they use teaching practices that focus on mathematics, leverage multiple mathematical competencies, affirm mathematical identities, challenge marginality, and draw on multiple resources of knowledge (Aguirre, Mayfield-Ingram, and Martin 2013). Furthermore, reasoning and sense making and the mathematical practices and habits of mind (NCTM 2009) are components of "doing mathematics."

## Developing Mathematical Agency

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The ways in which students participate in mathematics and express their mathematical identities determine their level of agency. *Agency* refers to the expression of one's identity (Murrell 2007). Students tell others through words and actions who they are and what their purpose is in a particular setting, space, or situation. In mathematics classrooms, agency is expressed in the ways that students engage in productive struggle, take risks to make their mathematical thinking visible, and understand that learning results when they successfully leverage an approach that works for them. In the Bike and Truck vignette, Jacobi and Charles made their mathematical reasoning visible by engaging in the classroom discussion to reach

agreement about a description of the movement of the truck. This level of engagement is an example of agency because both students verbalized their reasoning, persisted in explaining it, and remained engaged in the discussion until they reached an understanding of it. A high sense of agency allows and encourages students to continue with a rigorous course of study in mathematics.

Mathematical agency is about participating in mathematics in ways that are meaningful, both personally and socially (Berry 2016). Because participation in mathematics involves problem solving, reasoning, sense making, discourse, modeling, and appropriate use of tools, effective learning settings implement core equitable mathematics teaching practices that support mathematical agency (Bartell et al. 2017). Equitable mathematics teaching practices support identity and agency by creating structures for having students' mathematical ideas considered during instruction, supporting students in viewing themselves as having ownership of mathematical meaning, and coordinating enterprises across contexts to strengthen this ownership (Aguirre, Mayfield-Ingram, and Martin 2013; Oppland-Cordell and Martin 2015).

## Equitable Mathematics Teaching Practices

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Teachers build students' mathematical identities when they use teaching practices effectively to position students as mathematically competent by creating opportunities for them to demonstrate agency and efficacy (Wenger 1998). The eight Mathematics Teaching Practices articulated in *Principles to Actions* (NCTM 2014) provide a framework for making connections between these high-leverage teaching practices and the development of identity, agency; and competence. The eight Mathematics Teaching Practices are described below; connections are identified from each practice to identity and agency, and each description ends by suggesting what effective teachers do to make the connection:

- **Establish mathematics to focus learning.** Mathematics goals have two components: they describe the mathematical concepts, ideas, or methods that students will understand as a result of instruction, and they identify the mathematical practices that students will be learning (NCTM 2014). Establishing norms for participation involves creating structures to position each and every student as a full participant in mathematics and recognizing that participation builds agency (Turner et al. 2013). Within this teaching practice, teachers must establish classroom norms for participation that position each and every student as a competent mathematics thinker and valued mathematics contributor.
- **Implement tasks that promote reasoning and problem solving.** Effective use of mathematics tasks motivates learning and helps students build new mathematical knowledge through problem solving (NCTM 2014). Tasks that

require reasoning, problem solving, and modeling (i.e., tasks with high cognitive demand) result in a positive orientation toward mathematics and demonstrate that the student is a doer of mathematics (Boaler and Staples 2008). Within this teaching practice, teachers must use tasks in ways that develop positive dispositions toward mathematics and build students' mathematical identity.

- **Use and connect mathematical representations.** Mathematical representations are of particular importance in helping students to advance their understanding of mathematical concepts and procedures, make sense of problems, and engage in mathematical discourse (NCTM 2014). The use of multiple representations allows students to draw on multiple sources of knowledge (Boston et al. 2017). Drawing on multiple sources of knowledge acknowledges the mathematical, social, and cultural resources that students bring to mathematics. Teachers who use this teaching practice effectively validate the resources that students bring to mathematics and connect instruction with students' experiences and interests.
- **Facilitate meaningful mathematical discourse.** Discourse gives students opportunities to share ideas and clarify understanding, construct mathematical arguments, develop a language to express mathematical ideas, and learn the mathematical perspectives of others (NCTM 2014). Through discourse, students realize that their work and thinking serve an important role in the study of mathematics, thus positioning themselves and others as mathematically competent and reducing hierarchical status in mathematics classrooms (Boston et al. 2017). In implementing this teaching practice, teachers create structures that position students as mathematically competent and capable of sharing their mathematical thinking, connecting with peers to understand others' mathematical ideas, and participating in mathematical arguments.
- **Pose purposeful questions.** Purposeful questioning encourages students to explain and reflect on their own and others' thinking, and it allows teachers to discern what students know and understand and to use these insights to adapt lessons to meet the needs of students (NCTM 2014). Students who are consistently asked questions requiring them to explain their reasoning are positioned differently from students who are consistently asked questions not requiring explanation. The types of questions that students are asked can support positive mathematical identity and agency by positioning students as thinkers and doers of mathematics (Aguirre, Mayfield-Ingram, and Martin 2013). Teachers implementing this teaching practice pose purposeful questions and are mindful of which students they are asking which types of questions and whose ideas are privileged during discourse.
- **Build procedural fluency from conceptual understanding.** Conceptual understanding and procedural fluency are critical and connected components in the development of mathematical proficiency (NCTM 2014). Mathematics

instruction that focuses solely on remembering and applying procedures advantages students who are strong in memorization skills and disadvantages student who are not (Boston et al. 2017). Consequently, focusing primarily on the memorization of procedures may convey the message that mathematics is not about knowing and doing but instead is about memorizing. By contrast, teachers who make a practice of building fluency from conceptual understanding routinely connect conceptual understanding with procedural fluency so that students can make meaning of the mathematics and develop a positive disposition toward mathematics.

- **Support productive struggle in learning mathematics.** Teaching that embraces productive struggle provides opportunities for students to delve deeply into relationships among mathematical ideas and to develop understanding that leads them to apply their learning to new problem situations (NCTM 2014). This teaching practice involves giving students time to grapple with mathematical ideas (Hiebert and Grouws 2007). Grappling with ideas provides opportunities for students to develop a sense of agency by taking ownership of their mathematical thinking. Working within this teaching practice, teachers allow time for students to engage with mathematical ideas and provide supports through purposeful questioning to support perseverance and identity development.
- **Elicit and use evidence of student thinking.** Eliciting and using students' ideas require that teachers attend to more than just whether an answer is right or wrong. This teaching practice requires focusing on common patterns of reasoning and attending to how students understand a task and how ideas are developed over time (NCTM 2014). Whose ideas are elicited and used in the classroom has strong implications for mathematical identity and agency. Eliciting mathematical ideas from students who are perceived as always giving the right answer positions correctness as more valuable than mathematical thinking. Consequently, students may not share their thinking and may participate only when they believe that they may have a correct answer. By contrast, teachers who make a practice of eliciting and using evidence of students' mathematical thinking position each and every student as mathematically competent.

Figure 2 presents a crosswalk between the eight Mathematics Teaching Practices in *Principles to Actions* (NCTM 2014) and equitable mathematics teaching practices. The equitable mathematics teaching practices focus on practices supporting identity and agency. The recommendations in the crosswalk are not intended to be exhaustive; rather, the intent is to provoke ideas and serve as a first step for teachers who are intentional about implementing equitable teaching practices. Practices highlighted below draw on the work of Boston and colleagues (2017) and other researchers in mathematics education.

Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching	
Mathematics Teaching Practices	Equitable Teaching
<p><b>Establish mathematics goals to focus learning.</b> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</p>	<ul style="list-style-type: none"> <li>• Establish learning progressions that build students' mathematical understanding, increase their confidence, and support their mathematical identities as doers of mathematics.</li> <li>• Establish high expectations to ensure that each and every student has the opportunity to meet the mathematical goals.</li> <li>• Establish classroom norms for participation that position each and every student as a competent mathematics thinker.</li> <li>• Establish classroom environments that promote learning mathematics as just, equitable, and inclusive.</li> </ul>
<p><b>Implement tasks that promote reasoning and problem solving.</b> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</p>	<ul style="list-style-type: none"> <li>• Engage students in tasks that provide multiple pathways for success and that require reasoning, problem solving, and modeling, thus enhancing each student's mathematical identity and sense of agency.</li> <li>• Engage students in tasks that are culturally relevant.</li> <li>• Engage students in tasks that allow them to draw on their funds of knowledge (i.e., the resources that students bring to the classroom, including their home, cultural, and language experiences).</li> </ul>
<p><b>Use and connect mathematical representations.</b> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and to use as tools for problem solving.</p>	<ul style="list-style-type: none"> <li>• Use multiple representations so that students draw on multiple resources of knowledge to position them as competent.</li> <li>• Use multiple representations to draw on knowledge and experiences related to the resources that students bring to mathematics (culture, contexts, and experiences).</li> <li>• Use multiple representations to promote the creation and discussion of unique mathematical representations to position students as mathematically competent.</li> </ul>

Fig. 2. A crosswalk between the eight Mathematics Teaching Practices (NCTM 2014) and equitable mathematics teaching practices

Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching	
Mathematics Teaching Practices	Equitable Teaching
<p><b>Facilitate meaningful mathematical discourse.</b> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</p>	<ul style="list-style-type: none"> <li>• Use discourse to elicit students' ideas and strategies and create space for students to interact with peers to value multiple contributions and diminish hierarchical status among students (i.e., perceptions of differences in smartness and ability to participate).</li> <li>• Use discourse to attend to ways in which students position one another as capable or not capable of doing mathematics.</li> <li>• Make discourse an expected and natural part of mathematical thinking and reasoning, providing students with the space and confidence to ask questions that enhance their own mathematical learning.</li> <li>• Use discourse as a means to disrupt structures and language that marginalize students.</li> </ul>
<p><b>Pose purposeful questions.</b> Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.</p>	<ul style="list-style-type: none"> <li>• Pose purposeful questions and then listen to and understand students' thinking to signal to students that their thinking is valued and makes sense.</li> <li>• Pose purposeful questions to assign competence to students. Verbally mark students' ideas as interesting or identify an important aspect of students' strategies to position them as competent.</li> <li>• Be mindful of the fact that the questions that a teacher asks a student and how the teacher follows up on the student's response can support the student's development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.</li> </ul>

Fig. 2. *Continued*

Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching	
Mathematics Teaching Practices	Equitable Teaching
<p><b>Build procedural fluency from conceptual understanding.</b> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</p>	<ul style="list-style-type: none"> <li>• Connect conceptual understanding with procedural fluency to help students make sense of the mathematics and develop a positive disposition toward mathematics.</li> <li>• Connect conceptual understanding with procedural fluency to reduce mathematical anxiety and position students as mathematical knowers and doers.</li> <li>• Connect conceptual understanding with procedural fluency to provide students with a wider range of options for entering a task and building mathematical meaning.</li> </ul>
<p><b>Support productive struggle in learning mathematics.</b> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</p>	<ul style="list-style-type: none"> <li>• Allow time for students to engage with mathematical ideas to support perseverance and identity development.</li> <li>• Hold high expectations, while offering just enough support and scaffolding to facilitate student progress on challenging work, to communicate caring and confidence in students.</li> </ul>
<p><b>Elicit and use evidence of student thinking.</b> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</p>	<ul style="list-style-type: none"> <li>• Elicit student thinking and make use of it during a lesson to send positive messages about students' mathematical identities.</li> <li>• Make student thinking public, and then choose to elevate a student to a more prominent position in the discussion by identifying his or her idea as worth exploring, to cultivate a positive mathematical identity.</li> <li>• Promote a classroom culture in which mistakes and errors are viewed as important reasoning opportunities, to encourage a wider range of students to engage in mathematical discussions with their peers and the teacher.</li> </ul>

Fig. 2. *Continued*