Build procedural fluency from conceptual understanding Teacher and student actions, *continued*	
What are *teachers* doing?	**What are *students* doing?**
Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Striving to use procedures appropriately and efficiently.

Support Productive Struggle in Learning Mathematics

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Effective mathematics teaching supports students in struggling productively as they learn mathematics. Such instruction embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions. In contrast to productive struggle, unproductive struggle occurs when students "make no progress towards sense-making, explaining, or proceeding with a problem or task at hand" (Warshauer 2011, p. 21). A focus on student struggle is a necessary component of teaching that supports students' learning of mathematics with understanding (Hiebert and Grouws 2007). Teaching that embraces and uses productive struggle leads to long-term benefits, with students more able to apply their learning to new problem situations (Kapur 2010).

Discussion

In comparisons of mathematics teaching in the United States and in high-achieving countries, U.S. mathematics instruction has been characterized as rarely asking students to think and reason with or about mathematical ideas (Banilower et al. 2006; Hiebert and Stigler 2004). Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to "rescue" students by breaking down the task and guiding students step by step through the difficulties. Although well intentioned, such "rescuing" undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of the mathematics (Reinhart 2000; Stein et al. 2009). As teachers plan lessons, key components for them to consider are the student struggles and misconceptions that might

arise. Thinking about these in advance allows teachers to plan ways to support students productively without removing the opportunities for students to develop deeper understanding of the mathematics.

Mathematics classrooms that embrace productive struggle necessitate rethinking on the part of both students and teachers. Students must rethink what it means to be a successful learner of mathematics, and teachers must rethink what it means to be an effective teacher of mathematics. Figure 20 summarizes one such effort to redefine success in the mathematics classroom (Smith 2000), including expectations for students in regard to what it means to know and do mathematics, and actions for teachers with respect to what they can do to support students' learning, including acknowledging and using struggles as opportunities to learn.

Expectations for students	Teacher actions to support students	Classroom-based indicators of success
Most tasks that promote reasoning and problem solving take time to solve, and frustration may occur, but perseverance in the face of initial difficulty is important.	Use tasks that promote reasoning and problem solving; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students are engaged in the tasks and do not give up. The teacher supports students when they are "stuck" but does so in a way that keeps the thinking and reasoning at a high level.
Correct solutions are important, but so is being able to explain and discuss how one thought about and solved particular tasks.	Ask students to explain and justify how they solved a task. Value the quality of the explanation as much as the final solution.	Students explain how they solved a task and provide mathematical justifications for their reasoning.
Everyone has a responsibility and an obligation to make sense of mathematics by asking questions of peers and the teacher when he or she does not understand.	Give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.	Students question and critique the reasoning of their peers and reflect on their own understanding.
Diagrams, sketches, and hands-on materials are important tools to use in making sense of tasks.	Give students access to tools that will support their thinking processes.	Students are able to use tools to solve tasks that they cannot solve without them.
Communicating about one's thinking during a task makes it possible for others to help that person make progress on the task.	Ask students to explain their thinking and pose questions that are based on students' reasoning, rather than on the way that the teacher is thinking about the task.	Students explain their thinking about a task to their peers and the teacher. The teacher asks probing questions based on the students' thinking.

Fig. 20. Redefining student and teacher success. Adapted from Smith (2000, p. 382).

Teachers greatly influence how students perceive and approach struggle in the mathematics classroom. Even young students can learn to value struggle as an expected and natural part of learning, as demonstrated by the class motto of one first-grade math class: "If you are not struggling, you are not learning" (Carter 2008, p. 136). Teachers must accept that struggle is important to students' learning of mathematics, convey this message to students, and provide time for them to try to work through their uncertainties. Unfortunately, this may not be enough, since some students will still simply shut down in the face of frustration, proclaim "I don't know," and give up. Dweck (2006) has shown that students with a fixed mindset—that is, those who believe that intelligence (especially math ability) is an innate trait—are more likely to give up when they encounter difficulties because they believe that learning mathematics should come naturally. By contrast, students with a growth mindset—that is, those who believe that intelligence can be developed through effort—are likely to persevere through a struggle because they see challenging work as an opportunity to learn and grow.

The fixed mindset appears to be more prevalent in mathematics than in other subject areas (Dweck 2008). Mindsets, however, can be changed when students realize that they are in control of how they approach and view their own abilities to learn (Blackwell, Trzesniewski, and Dweck 2007). It is important to note that even students who have always gotten good grades may have a fixed mindset. These higher-achieving students are often concerned about how smart they appear to be, so they prefer tasks that they can already do well and try to avoid tasks in which they may make mistakes. Dweck (2008, p. 8) offers important words of caution:

> For the last few decades many parents and educators have been more interested in making students feel good about themselves in math and science than in helping them achieve. Sometimes this may take the form of praising their intelligence or talent and sometimes this may take the form of relieving them of the responsibility of doing well, for example, by telling them they are not a "math person." Both of these strategies can promote a fixed mindset.

A key message from this research is that teachers must acknowledge and value students for their perseverance and effort in reasoning and sense making in mathematics and must provide students with specific descriptive feedback on their progress related to these efforts (Clarke 2003; Hattie and Timperley 2007). This behavior by teachers may include giving feedback to students that values their efforts at trying varied strategies in solving problems, their willingness to ask questions about specific aspects of the task, or their attempts to be precise in explanations and use of mathematical language. For example, if students need to be more precise in their written or verbal explanations, the teacher could provide feedback that details how their explanations either are, or are not, precise. The result will be the development of students who are more likely to embrace difficulties and uncertainties as natural opportunities in solving problems and maintain engagement and persistence in their mathematics learning. (For an example of a warm-up routine that engages students in an eighth-grade classroom in productive struggle, view "My Favorite No: Learning from Mistakes" [https://www.teachingchannel.org/videos/class-warm-up-routine].)

Illustration

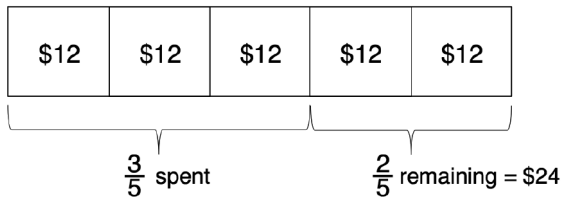
Figure 21 illustrates how two teachers, Ms. Flahive and Ms. Ramirez, present a real-world task related to fractions to two classes of fifth-grade students. In both classrooms, some students are immediately at a loss, upset, and vocal about their feeling that they don't know what to do. The two teachers respond to their students' discomfort in different ways.

Ms. Flahive and Ms. Ramirez teach fifth grade and plan their lessons collaboratively. Their current instructional unit focuses on fractions. They have selected the Shopping Trip task shown below because they think it will be accessible to their students yet provoke some struggle and challenge, since a solution pathway is not straightforward. The mathematics goal for students is to draw on and apply their understanding of how to build non-unit fractions from unit fractions and to use visual representations to solve a multi-step word problem:

> **Shopping Trip Task**
>
> Joseph went to the mall with his friends to spend the money that he had received for his birthday. When he got home, he had $24 remaining. He had spent $^3/_5$ of his birthday money at the mall on video games and food. How much money did he spend? How much money had he received for his birthday?

When Ms. Flahive and Ms. Ramirez present the problem in their classrooms, both teachers see students struggling to get started. Some students in both classrooms immediately raise their hands, saying, "I don't get it," or "I don't know what to do."

Ms. Flahive is very directive in her response to her students. She tells them to draw a rectangle and shows them how to divide it into fifths to represent what Joseph had spent and what he had left. She then guides her students step by step until they have labeled each one-fifth of the rectangle as worth $12, as shown below. Finally, she tells the students to use the information in the diagram to figure out the answers to the questions.



Ms. Ramirez approaches her students' struggles very differently. After she sees them struggling, she has them stop working on the problem and asks all the students to write down two things that they know about the problem and one thing that they wish they knew because it would help them make progress in solving the problem. Then Ms. Ramirez initiates a short class discussion in which several ideas are offered for what to do next. Suggestions include drawing a tape diagram or number line showing fifths, or just picking a number, such as $50 and proceeding through trial and error. Ms. Ramirez encourages the students to consider the various ideas that have been shared as they continue working on the task.

Fig. 21. Two teachers' responses to students' struggles to solve a multi-step word problem involving fractions

Ms. Flahive wants the students to be successful in figuring out the answer, so she begins to direct their work. Ms. Ramirez resists the temptation to step in but instead supports the students in considering what they know and what they need to figure out. As a result of these different approaches by the teachers to supporting struggling students, the students have very different opportunities to learn. Ms. Flahive's students learn that if you struggle and are vocal about your confusion, the teacher will ultimately tell you what to do; Ms. Ramirez's students learn that if you struggle and are at an impasse, the teacher will provide some assistance—but in the end you have to figure things out for yourself.

Teacher and student actions

Effective mathematics teaching uses students' struggles as valuable opportunities to deepen their understanding of mathematics. Students come to realize that they are capable of doing well in mathematics with effort and perseverance in reasoning, sense making, and problem solving. Teachers provide supports for students, individually and collectively, to work through uncertainties as they grapple with representing a mathematical relationship, explaining and justifying their reasoning, or finding a solution strategy for a mathematical problem. The table below summarizes teacher and student actions that embrace struggle as a natural aspect of learning in the mathematics classroom.

Support productive struggle in learning mathematics **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.